VOLUME 87 NO. IR2
JUNE 1961
PART 1

# JOURNAL of the Irrigation and Drainage Division

PROCEEDINGS OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

## BASIC REQUIREMENTS FOR MANUSCRIPTS

Original papers and discussions of current papers should be submitted to the Manager of Technical Publications, ASCE. Authors should indicate the technical division to which the paper is referred. The final date on which a discussion should reach the Society is given as a footnote with each paper. Those who are planning to submit material will expedite the review and publication procedures by complying with the following basic requirements:

1. Titles must have a length not exceeding 50 characters and spaces.

2. A summary of approximately 50 words must accompany the paper, a 300-word synopsis must precede it, and a set of conclusions must end it.

3. The manuscript (an original ribbon copy and two duplicate copies) should be double-spaced on one side of 8½-inch by 11-inch paper. Three copies of all illustrations, tables, etc., must be included.

4. The author's full name, Society membership grade, and footnote reference stating present employment must appear on the first page of the paper.

5. Mathematics are recomposed from the copy that is submitted. Because of this, it is necessary that letters be drawn carefully, and that special symbols be properly identified. The letter symbols used should be defined where they first appear, in the illustrations or in the text, and arranged alphabetically in an Appendix.

6. Tables should be typed (an original ribbon copy and two duplicate copies) on one side of 8½-inch by 11-inch paper. Specific illustrations and explanation must be made in the text for each table.

7. Illustrations must be drawn in black ink on one side of 8½-inch by 11-inch paper. Because illustrations will be reproduced with a width of between 3-inches and 4½-inches, the lettering must be large enough to be legible at this width. Photographs should be submitted as glossy prints. Explanations and descriptions must be made within the text for each illustration.

8. The desirable average length of a paper is about 12,000 words and the absolute maximum is 18,000 words. As an approximation, each full page of typed text, table, or illustration is the equivalent of 300 words.

9. Technical papers intended for publication must be written in the third person.

10. The author should distinguish between a list of "Reading References" and a "Bibliography," which would encompass the subject of his paper.

---



This Journal is published quarterly by the American Society of Civil Engineers. Publication office is at 2500 South State Street, Ann Arbor, Michigan. Editorial and General Offices are at 33 West 39 Street, New York 18, New York. $4.00 of a member's dues are applied as a subscription to this Journal. Second-class postage paid at Ann Arbor, Michigan.

The index for 1959 was published as ASCE Publication 1960-10 (list price $2.00); indexes for previous years are also available.

HW,IR,WW.

Vol. 87 No. IR 2

# Journal of the
# IRRIGATION AND DRAINAGE DIVISION
## Proceedings of the American Society of Civil Engineers



CONTENTS

June, 1961

Papers



(over)



Note.—Part 2 of this Journal is the 1961-21 Newsletter of the Irrigation and Drainage Division.

The three preceding issues of this Journal are dated September 1960, December 1960, and March 1961.

## DISCUSSION

2831 June, 1961 IR 2

Journal of the
IRRIGATION AND DRAINAGE DIVISION
Proceedings of the American Society of Civil Engineers

# FIELD MEASUREMENT OF HYDRAULIC CONDUCTIVITY[a]

By William W. Donnan,[1] F. ASCE, and V. S. Aronovici[2]

## SYNOPSIS

One of the major factors connected with the design of drainage systems is the measurement of hydraulic conductivity of the soil strata to be drained. This paper describes the work carried on to develop a device for this measurement. Laboratory studies resulted in the adoption of a small, brass, screen-type well point which could be inserted in the soil below the water table. The rate at which water was pumped out of the well point related directly to the conductivity of the strata being tested.

Field-type well points were then developed, using ordinary hardware pipe fitting. The field well points were then tested in the laboratory and in the field with good results.

This report describes how to make the field well points and also outlines a technique for field installation and operation of the device. In addition, graphs

Note.—Discussion open until November 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Irrigation and Drainage Division, Proceedings of the American Society of Civil Engineers, Vol. 87, No. IR 2, June, 1961.

a Contribution from Soil and Water Conservation Research Div., Agric. Research Service, U. S. Dept. of Agric.

1 Agric. Engr., Western Soil and Water Mgt. Research Branch, Soil and Water Conservation Research Div., Agric. Research Service, U. S. Dept. of Agric., Pomona, Calif.

2 Soil Scientist, Western Soil and Water Mgt. Research Branch, Soil and Water Conservation Research Div., Agric. Research Service, U. S. Dept. of Agric., Pomona, Calif.

are provided which relate quantity of water pumped under various head differentials with the corresponding hydraulic conductivity.

---

## INTRODUCTION

The Soil Conservation Service (SCS) has been called upon to design drainage systems as a part of their overall program of technical assistance to Soil Conservation Districts. One of the major factors connected with the design of any drainage system is the measurement of hydraulic conductivity of the soil strata to be drained. The design engineer must know how fast the drainage water will move to the drain in order to space the drains intelligently and to determine the speed with which the drains will dewater the soil profile. The SCS has listed this problem as one of its high priority research needs. Therefore, the objectives of this project were to develop a relatively simple device for use in measuring hydraulic conductivity rates of sand strata beneath the water table, to test this device in the field under operational conditions, and to set forth a methodology and technique which could be used in the field by field technicians.

## EXISTING METHODS AND TECHNIQUES

Various methods, devices, and techniques have been proposed for obtaining a measurement of hydraulic conductivity of soil strata in place. The most simple method is the single auger-hole method proposed originally by E. Diserens[3]. Subsequently, S. B. Hooghoudt[4] improved on earlier developments and later L. F. Ernst[5], using the relaxation method, developed an empirical formula which is claimed to be superior. Field techniques were developed by C. H. M. van Bavel and Don Kirkham[6]. W. C. Visser[7] later simplified and developed a field method which is now widely used in Europe. A detailed writeup of this technique has been completed by W. F. J. Van Beers[8].

Field methods other than the simple auger-hole method and involving the use of a piezometer have been proposed by Kirkham[9], Frevert and Kirkham[10],

---

3 "Beitrag zur Bestimmung der Durchlassigkeit des Bodens in naturlicher Bodenlagerung," by E. Diserens, Schweiz, Landw. Monatshefte, Vol. 12, 1935, pp. 188-198, 204-212.

4 "Bijdragen tot de Kennis von eenige natuurkundige grootheden van den Grond," by S. B. Hooghoudt, 4 Versl. Landb. ond., Vol. 42(13)B, 1936, pp. 449-541.

5 "En nieuwe Formule voor de Berekening van de Doorlaatfactor met de Boogatenmethode," by L. F. Ernst, Rap. Landbouwproefsta en Bodemkundig, Inst. T.N.O., Groningen, 1950.

6 "Field Measurement of Soil Permeability using Auger Holes," by C. H. M. van Bavel and Don Kirkham, Proceedings, Soil Science Soc. of America, Vol. 13, 1949, pp. 90-96.

7 "Title Drainage in the Netherlands," by W. C. Visser, Netherlands Journal of Agricultural Science, Vol. 2, 1954, pp. 69-87.

8 "Auger Hole Method," by W. F. J. Van Beers, Bulletin No. 1, Internatl. Inst. for Land Reclamation and Improvement, Wageningen, Netherlands, 1958.

9 "Proposed Method for Field Measurement of Permeability of Soil Below a Water Table," by Don Kirkham, Proceedings, Soil Science Soc. of America, Vol. 10, 1946, pp. 58-68.

10 "A Field Method for Measuring the Permeability of Soil Below a Water Table," by R. K. Frevert and Don Kirkham, Proceedings, Highway Research Bd., Vol. 28, 1948, pp. 433-442.

Luthin and Kirkham[11], and Reeves and Kirkham[12]. In addition, E. C. Childs[13] proposed the use of a two-well method, where water is pumped out of one well and into an adjacent well. Subsequently, this technique was improved upon by utilizing three-well and four-well methods.

Nearly all of these techniques, while valid, are of necessity rather cumbersome. They require a considerable degree of exact methodology to achieve accurate results. The method after Visser[7] has been, so far, the most easily adaptable to field problems. This method has been tentatively adopted by the SCS.

## PROGRESS OF THE WORK

All of the known techniques for measuring hydraulic conductivity of the soil strata in the field have one common weakness. This is that the "A" function, or area and geometric shape of flow into the auger hole, piezometer, well, or other device is difficult to determine. Furthermore, once it is determined, this factor is difficult to maintain under most field conditions. It is a well-known fact that flow of water into an open cavity in the soil creates sloughing which, in turn, alters or changes the size and shape of the cavity. The present work to date (1961) has been aimed at developing an instrument with a fixed or predetermined "A" function, thus eliminating this troublesome variable from the technique.

The initial objective also was to develop a relatively economical unit which could be readily purchased or made without resorting to elaborate laboratory or machine shop construction. Therefore, the first devices developed and tested were extreme in their simplicity. They consisted of ceramic and sintered glass airstones.

Fig. 1 shows a picture of three of the units tested originally. The left-hand device in Fig. 1 is a sintered glass gas diffuser; the center device is a ceramic fish tank airstone. The technique used was to insert these devices in soil to be tested and to apply a low head suction to the outlet tube. It was quickly apparent the airstone was not applicable because of the tendency to clog.

A series of tests were conducted using filter paper wrapped around the airstones, but still the results were not satisfactory. The airstones tended to age and over a period of several days the stones themselves would decrease in permeability even in clear water.

## WELL POINTS

The next step was to construct some type of a brass screen well point which would have a fixed or constant "A" function and which could be used in place of an ordinary piezometer or auger hole. Several experimental well points were tested. These included double core typed wherein the inner core could be

11 "Piezometer Method for Measuring Permeability of the Soil in Situ Below a Water Table," by J. N. Luthin and Don Kirkham, Soil Science, Vol. 68, 1949, pp. 349-358.

12 "Soil Anisotropy and Some Field Methods for Measuring Permeability," by R. C. Reeve and Don Kirkham, Transactions, Amer. Geophysical Union, Vol. 32, 1951, pp. 582-590.

13 "Measurement of Hydraulic Permeability of Saturated Soil in Situ. 1. Principles of a Proposed Method," by E. C. Childs, Proceedings, Royal Soc. of London, Series A, Vol. 215, 1952, pp. 525-535.

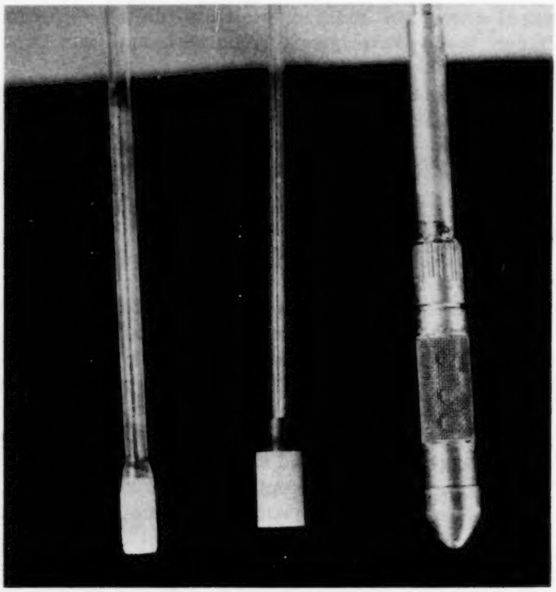


FIG. 1.—WELL-POINT DEVICES USED IN HYDRAULIC CONDUCTIVITY TESTS

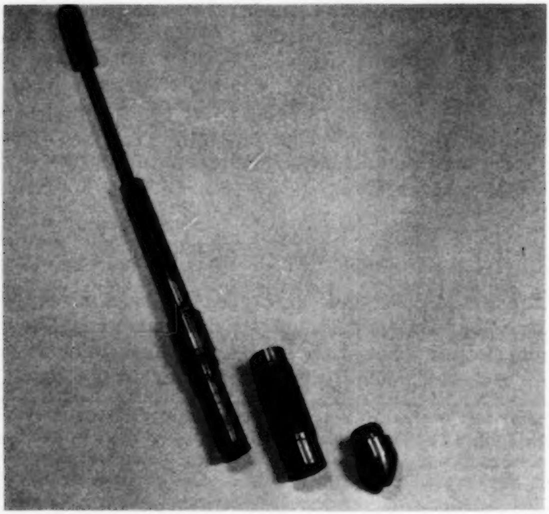


FIG. 2.—NON-CLOGGING BRASS SCREEN WELL-POINT

wrapped with filter paper. The final design, and the one with which most of the subsequent laboratory testing was carried out, is shown as the right-hand device in Fig. 1. Fig. 2 shows this well point, unassembled.

The test well point was constructed of brass and consisted of an inner perforated brass tube with an outer cage assemble and demountable brass tip. The outer cage is covered with 40-mesh brass screen. The test point has a screen length of 3.5 cm (1.375 in.), an outside diameter at the screen of 1.85 cm (0.728 in.), and an overall intake area of 20.29 sq cm (3.146 sq in.).

The "A" function for this device was originally computed after Luthin and Kirkham[11]. They evaluated the "A" function by means of a three-dimensional electrical analogue of the ground-water problem. However, this work was done

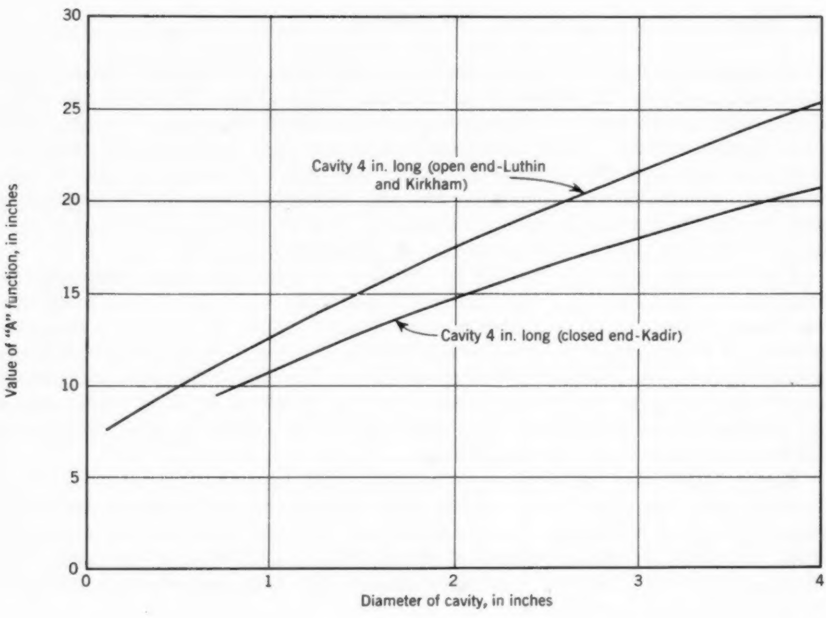



FIG. 3.—GRAPHS FOR COMPUTING "A" FUNCTION

using a cavity with an open-bottom end whereas the test well point has a closed end. Subsequent investigation by N. A. Kadir[14] revealed the existence of the desired "A" function. Kadir developed data for computing the "A" function values for sealed bottom cavities. The Kadir cavity more nearly approaches the geometric boundaries of the Pomona well point. Thus, the Kadir curve has been used to compute "A" function based on data by Luthin and Kirkham[11], and by Kadir[14].

14 "Measurement of Permeability of Saturated Soils Below the Water Table," by N. A. Kadir, Ph.D. Dissertation, Utah State Univ., Logan, Utah, 1955.

An example of how to compute the "A" function is given as follows: The ratio of length to diameter of the brass screen device is computed to be 1.0 to 0.529. The graph in Fig. 3 for computing the value of the "A" function is drawn for a 4-in. length cavity. Thus, a corresponding diameter to maintain geometric similarity would be 4.0 x 0.529 in. = 2.15 in. diameter. Referring to Fig. 3, the "A" function value after Kadir would be interpolated as 15.3 in. for a 4-in. length device. The value, then, for the brass screen device of 1.375-in. length would be (1.375/4) 15.3 = 5.25 in. This value has been used for all computations involving the brass screen well point. The value is assumed to be valid for any measurement of hydraulic conductivity made at a reasonable depth below the water table, say 5 in. or more below, and for a reasonable height above an impervious layer, say 5 in. or more above.

## TEST TANK

The laboratory tests of the brass screen well point were conducted in a tank which was built and installed at the Pomona Laboratory. It was 30 cm wide, 45 cm high, and 95 cm long. The front face was fitted with a 1-cm thick, clear plastic plate. Water supply reservoirs were provided along the full width and height of each and with access of water into the entire soil profile through screened access holes. Provision was also made for input and withdrawal of water from the bottom of the tank through a 3/4-in. diameter perforated pipe positioned along the floor of the tank and outletted through one end.

The tank was filled to a depth of 40 cm with pit-run dune sand (classified as Hanford fine sand) having a high degree of isotropic hydraulic characteristics. Two manometers were provided by embedding ceramic airstones at approximately the third points within the tank. These airstones were fitted with tubes and brought to manometers on the front face of the tank. The well point itself was installed in the approximate center of the tank at a depth of 20 cm below the surface of the sand in the tank. Installation was made by pushing the well point vertically into the saturated sand.

Suction head was introduced to the well point by means of a siphon which could be adjusted to provide various head differentials for various test runs. Water levels in the tank and suction head on the well point were both measured from the same meter stick affixed to the outside front face of the tank. The suction head was reflected in a glass manometer arrangement along the side of the meter stick.

## TEST RUNS

In making a test run of the well point, the sand in the tank was completely saturated by the introduction of water from the floor intake tube. The initial test runs were not made until the tank has been saturated and dewatered to field capacity through a number of cycles. This allowed for a reasonable degree of settling and repose of the sand prior to actual well-point tests.

The water in the tank was then brought to a ponded depth of approximately 1 cm above the sand and maintained at this depth throughout the test run. During a test run, water was introduced from the end supply tanks fed through a Mariotte arrangement. The pond surface was held at a fixed level by means of a strong suction outlet suspended at the desired pond level.

A slight suction was then applied to the well-point tube and all air bubbles were removed from the siphon. Water was allowed to flow out of the tank through the siphon with the well point acting as the intake source. The volume of flow through the well point then became a function of suction head as provided by the siphon tube. Since the tank was mounted on top of a 4-ft-high laboratory bench, the siphon arrangement provided for a convenient range of suction head from 0.1 cm to about 80 cm.

Volume of flow coming from the siphon was measured by taking periodic, short-duration (1-min to 10-min) readings using a timing device. The volume was measured in a suitable graduate beaker reading to 5 cu cm. No temperature corrections were made since no attempt was made to provide continuity between tests fro hour to hour or day to day.

TABLE 1.—HYDRAULIC CONDUCTIVITY OF HANDFORD FINE SAND AS MEASURED BY LABORATORY-PACKED 3-IN. CORES

| Cylinder No. | Mariotte | | | Time | | | Hydraulic conductivity, in inches per hour |
|---|---|---|---|---|---|---|---|
| | $q_1$, in ml | $q_2$, in ml | Q, in ml | $t_1$, in minutes | $t_2$, in minutes | T, in minutes | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| 204 | 175 | 495 | 320 | 3.85 | 10.20 | 6.35 | 18.6 |
| 204 | 0 | 500 | 500 | 10.94 | 20.96 | 10.02 | 18.3 |
| 204 | 0 | 500 | 500 | 21.70 | 31.48 | 9.78 | 18.9 |
| 204 | 0 | 500 | 500 | .12 | 10.50 | 10.38 | 17.8 |
| 204 | 0 | 500 | 500 | 11.60 | 21.80 | 10.20 | 18.1 |
| 204 | 75 | 500 | 425 | 28.40 | 37.35 | 8.95 | 17.5 |
| 252 | 325 | 500 | 175 | 4.14 | 6.96 | 2.82 | 22.9 |
| 252 | 10 | 500 | 490 | 7.60 | 15.79 | 8.19 | 22.1 |
| 252 | 0 | 500 | 500 | 16.40 | 24.98 | 8.58 | 21.5 |
| 252 | 10 | 500 | 490 | 25.53 | 33.96 | 8.43 | 21.5 |
| 252 | 50 | 500 | 450 | .46 | 8.38 | 7.92 | 21.0 |
| 252 | 0 | 500 | 500 | 17.50 | 26.55 | 9.05 | 20.4 |
| 252 | 0 | 500 | 500 | 28.10 | 37.50 | 8.60 | 21.5 |
| 201 | 250 | 505 | 255 | 4.37 | 10.48 | 6.11 | 15.4 |
| 201 | 25 | 500 | 475 | 11.58 | 23.18 | 11.60 | 15.1 |
| 201 | 5 | 500 | 495 | 23.67 | 35.45 | 11.78 | 15.4 |
| 201 | 50 | 500 | 450 | .86 | 12.48 | 11.62 | 14.3 |
| 201 | 0 | 500 | 500 | 13.46 | 26.65 | 12.19 | 15.2 |
| 201 | 0 | 350 | 350 | 28.82 | 38.24 | 9.42 | 13.7 |

The hydraulic conductivity of the Hanford fine sand in the tank was checked by the use of laboratory-packed cores. These were the conventional 3-in. diameter cores 3 in. long. Table 1 shows data from a series of permeameter tests of soil cores. The apparent hydraulic conductivity of this sand is between 15 in. and 20 in. per hr.

## RESULTS

Data from the earlier tests of the brass well point in the Hanford sand indicated that considerable head loss was being manifested in the apparatus used

to provide and record suction to the well point. To circumvent this and to make a head loss measurement, an auxiliary manometer was installed inside the well point itself and outletted via suitable tubes to the front face of the tank. This provided for the simultaneous recording of both the siphon tube manometer and the apparent suction head, as manifested by the water level in the well-point manometer. Table 2 is a typical example of test runs with the well point.

TABLE 2.—OBSERVED DATA FROM BRASS WELL-POINT TESTS

| Trial No. | $h_1$, in cm[a] | $h_2$, in cm[b] | $h_1 - h_2$ in cm | $h_3$, in cm[c] | $h_1 - h_3$, in cm | T, in minutes | Q, in ml | Q/T, in ml | Hydraulic Conductivity: Siphon-tube manometer, in inches per hour | Hydraulic Conductivity: Well-point manometer, in inches per hour |
|---|---|---|---|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) |
| 1 | 9.0 | 17.0 | 8.0 | 15.3 | 6.3 | 6.0 | 310 | 51.7 | | |
| 2 | 9.0 | 17.0 | 8.0 | 15.3 | 6.3 | 6.0 | 305 | 50.8 | 11.48 | 14.65 |
| 3 | 9.0 | 17.0 | 8.0 | 15.3 | 6.3 | 5.0 | 255 | 51.0 | | |
| 4 | 9.0 | 25.0 | 16.0 | 21.3 | 12.3 | 4.0 | 385 | 96.2 | | |
| 5 | 9.0 | 25.0 | 16.0 | 21.3 | 12.3 | 4.0 | 385 | 96.2 | 10.70 | 13.81 |
| 6 | 9.0 | 25.0 | 16.0 | 21.3 | 12.3 | 3.0 | 280 | 93.5 | | |
| 7 | 9.0 | 35.0 | 26.0 | 28.5 | 19.5 | 3.5 | 525 | 150.0 | | |
| 8 | 9.0 | 35.0 | 26.0 | 28.5 | 19.5 | 3.0 | 450 | 150.0 | 10.38 | 13.85 |
| 9 | 9.0 | 35.0 | 26.0 | 28.5 | 19.5 | 3.0 | 450 | 150.0 | | |
| 10 | 9.0 | 47.0 | 38.0 | 36.6 | 27.6 | 2.0 | 420 | 210.0 | | |
| 11 | 9.0 | 47.0 | 38.0 | 36.6 | 27.6 | 2.0 | 525 | 210.0 | 9.94 | 13.65 |
| 12 | 9.0 | 47.0 | 38.0 | 36.6 | 27.6 | 2.0 | 420 | 210.0 | | |
| 13 | 9.0 | 55.0 | 46.0 | 42.0 | 33.0 | 2.0 | 500 | 250.0 | | |
| 14 | 9.0 | 55.0 | 46.0 | 42.0 | 33.0 | 2.0 | 500 | 250.0 | 9.85 | 13.62 |
| 15 | 9.0 | 55.0 | 46.0 | 42.0 | 33.0 | 2.0 | 500 | 250.0 | | |
| 16 | 9.0 | 75.0 | 66.0 | 54.8 | 45.8 | 1.5 | 500 | 345.0 | 9.40 | 13.50 |
| 17 | 9.0 | 55.0 | 46.0 | 42.1 | 33.1 | 2.0 | 480 | 240.0 | 9.40 | 13.10 |
| 18 | 9.0 | 47.0 | 38.0 | 37.3 | 28.3 | 2.0 | 400 | 200.0 | 9.50 | 12.72 |
| 19 | 9.0 | 35.0 | 26.0 | 29.0 | 20.0 | 3.0 | 415 | 138.2 | 9.57 | 12.43 |
| 20 | 9.0 | 25.0 | 16.0 | 21.8 | 12.8 | 3.0 | 265 | 88.4 | 9.94 | 12.41 |
| 21 | 9.0 | 17.0 | 8.0 | 15.4 | 6.4 | 3.0 | 132 | 44.0 | 9.55 | 12.39 |

[a] Water surface
[b] Siphon-tube manometer
[c] Well-point manometer

Note the apparent discrepancy between the hydraulic conductivity as determined for soil cores and the conductivity as computed from the well point test. It should also be noted that when the applied head was used in successive tests on a descending scale of magnitude (trials 16 to 21), the comparison between successive conductivities was more favorable. In other words, if the well point was initially stressed, succeeding trials with differential heads did not vary appreciably.

This phenomenon prompted a series of tests wherein a given suction head was applied and allowed to run over an extended period of several hours. Periodic readings were made to record drift or fluctuations in quantity of flow. It was found that after about 30 min the flow came into equilibrium. Subsequent tests were made wherein the well point was stressed at some higher given suction head and then the head was lowered to the desired amount. The quantity

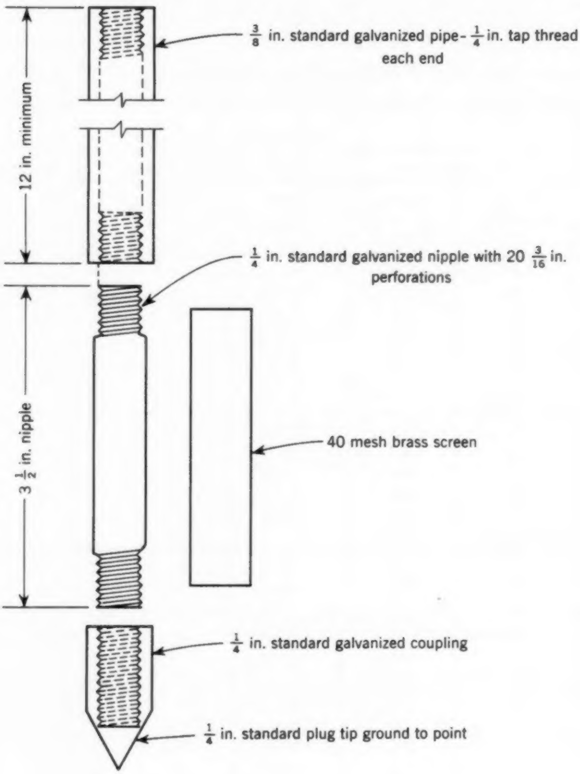



FIG. 4.—POMONA WELL POINT

of flow immediately reached equilibrium. This suggested a technique which was found to be advantageous when the device was tested in the field.

Again referring to Table 2, it is apparent that excessive suction heads created excessive head losses. These occurred not only in the siphon manometer observations but also to a degree in the well point manometer observations. This circumstance, while not a factor in actual field applications, does suggest the use of low head differentials in field use. Low head differentials would tend to minimize the head losses which will be a characteristic of the flow of water

through the brass screen of the well point. At present, the recommendations for field use call for only a 3-in. or 4-in. head differential to be applied to the well point.

## FIELD WELL POINTS

The primary goal of this project was to developa simple instrument for use by SCS technicians and others. Thus an attempt was made to construct a simple adaptation of the previously described and tested brass screen. The field well points consist of ordinary galvanized pipe nipples, couplings, and plugs and are constructed as follows: Cut a 6-ft length of 3/8-in. galvanized pipe and tap thread the inside ends with a 1/4-in. standard pipe tap. For the inner core, use a 3 1/2-in. standard galvanized pipe nipple 1/4-in. size and bore 20 3/16-in. holes in it, suitably spaced. The nipple is threaded into one end of the

TABLE 3.—OBSERVED DATA FROM FIELD-TYPE WELL POINT TESTS

| Sand type | Hydraulic Conductivity | | | | | |
|---|---|---|---|---|---|---|
| | 3-in core tests | | 150-lb drum tests | | Well point tests | |
| | Core number | inches per hour | Date tested | inches per hour[a] | Date tested | inches per hour[a] |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| Newport Beach sand | 1 | 75 | 7/18 | 78 | 7/20 | 58 |
| | 2 | 74 | 7/19 | 75 | 7/21 | 60 |
| | 3 | 76 | 7/20 | 68 | 7/22 | 61 |
| | 4 | 75 | 7/21 | 63 | 7/25 | 60 |
| | 5 | 75 | 7/22 | 60 | 7/26 | 62 |
| | | | 7/23 | 56 | | |
| | Average | 75.0 | | 66.7 | | 60.2 |
| Alberhill Quartz sand | 1 | 44 | 7/18 | 59 | 7/20 | 59 |
| | 2 | 56 | 7/18 | 57 | 7/21 | 64 |
| | 3 | 58 | 7/20 | 58 | 7/22 | 62 |
| | 4 | 49 | 7/21 | 59 | 7/25 | 59 |
| | 5 | 44 | 7/26 | 57 | 7/26 | 57 |
| | Average | 50.2 | | 58.0 | | 60.2 |
| San Antonio Wash sand | 1 | 12 | 7/15 | 15 | 7/20 | 15 |
| | 2 | 15 | 7/18 | 13 | 7/21 | 18 |
| | 3 | 17 | 7/20 | 14 | 7/22 | 14 |
| | 4 | 15 | 7/21 | 17 | 7/25 | 17 |
| | 5 | 17 | 7/26 | 17 | 7/26 | 18 |
| | Average | 15.2 | | 15.2 | | 16.5 |
| Etiwanda Blow sand | 1 | 17 | 7/18 | 25 | 7/20 | 30 |
| | 2 | 17 | 7/17 | 25 | 7/21 | 30 |
| | 3 | 17 | 7/20 | 24 | 7/22 | 26 |
| | 4 | 18 | 7/21 | 24 | 7/25 | 30 |
| | 5 | 18 | 7/22 | 25 | 7/26 | 28 |
| | Average | 17.2 | | 24.6 | | 28.8 |

[a] Value is arithmetic average of 10 individual trials as shown in Table 4.

6-ft pipe. For the screen, use a 3-in. by 2-in. square of 40-mesh brass screen and solder it into a cylinder 3 in. long. This screen slips loosely over the nipple core.

For the tip-end component, use a standard 1/4-in. galvanized pipe coupling with a standard 1/4-in. pipe plug threaded on one end. The end peice threads onto the end of the nipple and should be threaded up snug against the cylindrical brass screen. The blunt tip and plug can be shaped to a conical point. The

TABLE 4.—WELL-POINT TESTS OF HYDRAULIC CONDUCTIVITY

| Trial No. (1) | Applied head, in cm (2) | Quantity pumped, in cu cm (3) | Elapsed time, in minute (4) | Rate of flow, in cu cm per min (5) | Hydraulic conductivity, in inches per hour (6) |
|---|---|---|---|---|---|
| 1 | 10 | 300 | 2.69 | 111.5 | 13.8 |
| 2 | 10 | 400 | 3.64 | 109.8 | 13.6 |
| 3 | 10 | 200 | 1.80 | 111.0 | 13.7 |
| 4 | 10 | 200 | 1.84 | 108.5 | 13.5 |
| 5 | 10 | 300 | 2.70 | 111.0 | 13.7 |
| 6 | 10 | 300 | 2.72 | 110.0 | 13.6 |
| Low Vacuum[a] | | | | | |
| 1 | 10 | 300 | 2.91 | 103.0 | 12.8 |
| 2 | 10 | 450 | 4.42 | 101.0 | 12.5 |
| 3 | 10 | 250 | 2.45 | 102.0 | 12.7 |
| 4 | 10 | 250 | 2.46 | 101.5 | 12.6 |
| High Vacuum[b] | | | | | |
| 1 | 10 | 300 | 2.75 | 109.0 | 13.5 |
| 2 | 10 | 300 | 2.74 | 109.5 | 13.6 |
| 3 | 10 | 200 | 1.86 | 107.5 | 13.3 |
| 4 | 10 | 200 | 1.78 | 112.5 | 13.9 |
| 5 | 10 | 400 | 3.64 | 109.7 | 13.6 |

[a] In tests carried out under a low vacuum, the pump used to remove water from inside the well point was set at a speed which just barely kept ahead of the inflow to the well point.

[b] In tests carried out under a high vacuum, the pump used to remove water from inside the well point was set at a speed which pumped considerable quantities of air along with the inflow water to the well point.

field-type well point costs about $1.25 and fits smoothly through the end of a standard Viehmeyer King tube. See Fig. 4 for a sketch of this device.

## TESTS OF FIELD WELL POINTS

The field-type well point has been tested extensively in the laboratory. The technique used was to pack approximately 150 lb of sand material into 4-ft-high steel drums. The drums were provided with water inflow and outflow pipes and were fitted with manometers so that they could be used as a permeameter. Thus the sand material in the drums could be tested for hydraulic conductivity in situ.

In testing the well point, the device was shoved vertically into the sand and clamped into position with the well point about 12 in. from the surface. A suction tube was then lowered inside the well point shaft. The end of this tube was placed 3 in. below the ponded water surface. When suction was applied, a 3-in. hydraulic head was exerted on the well point. Suction was provided with a suitable vacuum and quantity of flow was measured. From these data the hydraulic conductivity could be computed and a comparison made with the conductivity as measured by the 150 lb, 4 ft high drum, permeameters. A third comparison was obtained by preparing small core samples of the sand material and testing them for hydraulic conductivity in the 3-in. permeameters.

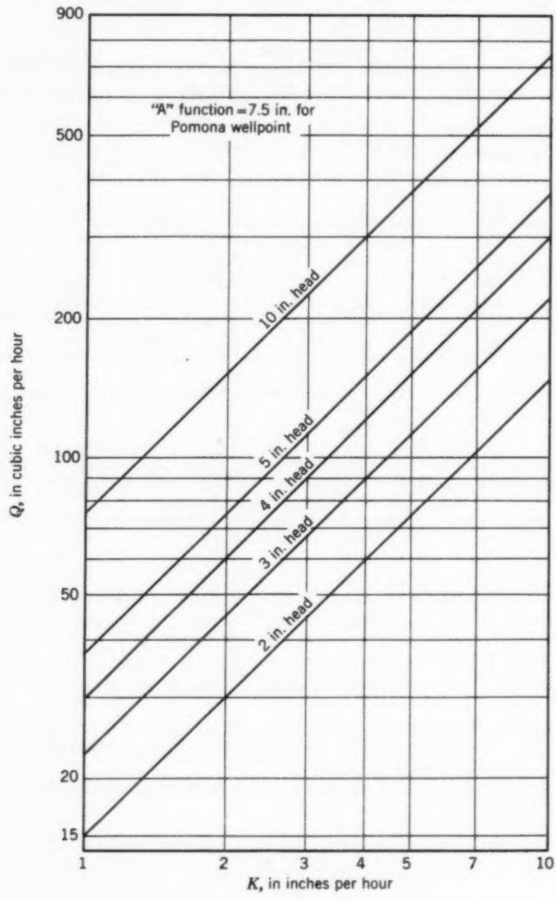



FIG. 5.—RATIO BETWEEN q AND K

Four different types of sands, having a range of hydraulic conductivity from approximately 15 in. per hr up to 75 in. per hr, were used in these tests. Table 3 shows the results. Table 4 is a series of observations made in the laboratory using the well point in a Hanford fine sand.

To compute hydraulic conductivity, an adaptation of the well-known Darcy Law of the flow of water through a saturated porous media has been used.

$$Q = K I A \qquad (1)$$

or

$$K = \frac{Q}{LA} \qquad (2)$$

in which K is the hydraulic conductivity; Q denotes the quantity of water pumped; I represents the hydraulic gradient; and A is the area of flow.

The adaptation as developed by Kirkham is as follows:

$$K = \frac{Q}{(\text{"A" function})\ (\text{Head})} \qquad (3)$$

in which K is the hydraulic conductivity; Q represents the quantity of water pumped; (Head) is the suction head applied; and ("A" function) is a value based on the Kadir data for closed-end cavities.

Fig. 5 shows the relationship between the quantity of water pumped for various suction heads applied and the corresponding hydraulic gradients. These curves have been computed for use with the field-type Pomona well point, having the same geometric configuration as has been previously described and having a computed "A" function value after Kadir of 7.5 in.

## FIELD TECHNIQUES

Installation of the well point in the field can be made by driving the soil tube to the approximate depth where a measurement of hydraulic conductivity is to be made. Then, by inserting the well point in the empty soil tube, both devices are lowered into the hole. The well point is then pushed on down 6 in. or 8 in. beyond the soil tube into the stratum where a measurement is to be made.

The water table is allowed to come into equilibrium and is measured. Then a small-diameter suction tube is lowered inside the well point pipe to a point 3 in. below the water table. By pumping or providing a suction to this tube, a 3-in. hydraulic head is applied to the well point. Pumping can be done either by an attachment to the vacuum on a pickup truck or by an ordinary Ford fuel pump.

A Ford fuel pump can be fixed to a 2-in. by 6-in. board and can be activated by hand. This device produces a 9-ft to 10-ft vacuum which will suffice for most field observations. The quantity of water pumped in a given period of time is determined. Usually it is best to pump for about 5 min to 10 min and then take several short-term readings of rate of flow. Using the curves in Fig. 5, the hydraulic conductivity can be computed by dirct ratio.

## CONCLUSIONS

A small, screened well point has been developed for use in making measurments of the hydraulic conductivity of sand strata below a water table. This device has been tested in the laboratory and in the field with good results.

Journal of the
IRRIGATION AND DRAINAGE DIVISION
Proceedings of the American Society of Civil Engineers

# OUTLOOK FOR ECONOMIC USE OF FRESH WATER FROM THE SEA[a]

By Samuel B. Morris,[1] F. ASCE

## SYNOPSIS

The need and market for fresh water from the sea or from inland saline waters are mainly dependent on the cost of producing such water, storing, and transmitting it to places of use in competition with other sources of fresh water supply.

Foreseeable costs of conversion far exceed prevailing costs of major domestic, industrial, and irrigation water supplies. Typical costs are presented for municipal, industrial, and irrigation supplies and for current and foreseeable costs of fresh water conversion. Comparison is made of water availability and requirements in the humid eastern United States and the semiarid western United States.

Conversion costs are too high for major sources of supply but are acceptable for isolated plants and, possibly in the future, for larger general use on the western Gulf coast of Texas, which will ultimately be short of water. Con-

Note.—Discussion open until November 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Irrigation and Drainage Division, Proceedings of the American Society of Civil Engineers, Vol. 87, No. IR 2, June, 1961.

[a] Presented at the June 22, 1960 ASCE Convention in Reno. Nev.

[1] Cons. Engr., Los Angeles, Calif.

version of inland saline waters is further faced with expensive disposal of wastes, a subject that should be under study in parallel with desalinization research.

---

## INTRODUCTION

The need and market for fresh water from the sea or from inland saline waters are mainly dependent on the cost of producing such water, storing, and transmitting it to places of use in competition with other sources of fresh water supply. In fact, even if no fresh water is available as a competing factor, high cost of such fresh water from the sea or from saline waters in any given area may not justify an industry, community, or agriculture in locating in such area. Conversely, other conditions may dictate the use of a given site for mining, industrial, or military purposes almost irrespective of the cost of water necessary to the establishment and its operating personnel. Quite obviously, these latter conditions affect special limited sites and developments and are not in the nature of general urban development or irrigated agriculture.

The purpose of this paper will be to consider the costs of normal water supplies in comparison to present and anticipated future costs of producing fresh water from the sea and from saline waters; then, to relate this cost comparison to possible needs within the continental United States, except Alaska.

## WATER NEEDS ARE DEPENDENT ON WATER COSTS

The need for water is not independent of cost, especially for irrigation use. If adequate high quality water costs only $1 per acre-ft, all semiarid lands of proper elevation, topography, and soil type will be irrigated when markets for their crops develop. However, if irrigation water costs as much as $20 per acre-ft (6.14 cents per 1000 gal), then only favored lands and locations can grow sufficiently high-value crops, such as cotton, citrus, fruit, nut, and truck gardens, to pay out. In general, municipal water-users have been able to pay whatever the water costs. The cost and character of water use, however, become an important factor in the location of an industry requiring large use of water in relation to the market value of its products.

*Water Costs.*—Costs of irrigation water delivered to the farmer's headgate are generally of the order of $1 to $10 per acre-ft (0.307 - 3.07 cents per 1000 gal) and may go as high as $40 per acre-ft (12.28 cents per 1000 gal) for limited areas of high-value crops.

While retail costs of municipal water delivered through the customer's meter generally vary from 10 to 30 cents per 1000 gal ($32.60 - $97.80 per acre-ft) the water production costs are generally 3.07 to 6.14 cents per 1000 gal ($10 - $20 per acre-ft) for local supplies and 6.14 to 21.49 cents per 1000 gal ($20 to $70 per acre-ft) for present (1961) and planned water supplies conveyed hundreds of miles through large aqueducts.

Senate Select Committee on National Water Resources Reports on Water Costs.—It will be helpful to examine the prices that are being paid for water

as reported to the Select Committee on National Water Resources, United States Senate.[2]

> "With respect to the cost of water for industrial use, there is a wide variation. The major water-using establishments usually self-supply their requirements, attempting to hold the cost within the range of 5 to 10 cents per 1,000 gal ($16.30 to $32.60 per acre-ft). On the other hand, the smaller volume water-using establishments which average only from 4% to 16% as much water, usually purchase from a public water utility at costs ranging from 12 to 20 cents per 1,000 gal ($39.12 to $65.20 per acre-ft) for larger volumes and from 20 to 28 cents per 1,000 gal ($65.20 to $91.28 per acre-ft) for the extremely small volume users. No comprehensive assembly has been made of these costs, but numerous articles in technical magazines have cited instances."

In the foregoing quotation the writer has inserted prices per acre-ft to conform to irrigation usage.

These prices obviously are waterworks sale prices and not water production costs. Production costs are generally not over one-fourth to one-third of the sale price. Applying 1/3 as a factor, water production costs to the utility would indicate for "smaller volume water-using establishments," $13.04 to $21.73; and for "extremely small volume users," $21.73 to $30.43 per acre-ft. These figures indicate the quoted cost to major water-using establishments is out of line for their own self-supplies which must be materially lower than prices paid to public utilities. Experience has shown that major users generally obtain their fresh water supplies at $5 to $10 per acre-ft.

The quotation[2] continues:

> "The actual consumption or depletion of water as a resource is indicated in the 1954 Census of Manufacturers in the difference between the intake and the discharge of the manufacturing industries. In aggregate, such depletion is less than 10% of the total intake."

In the electrical utility industry, accurate data of kilowatt demand and kilowatt hour produced and sold are available for the entire industry in the weekly reports of the Federal Power Commission (FPC). It is unfortunate that there is an absence of such accurate reporting of water production, water sales, and consumptive use of water. It is recognized that such data like hydrologic data are more difficult to procure, and involve intelligent estimates from incomplete information. However, other data are available. For example, tremendous quantities of cooling water and other nonconsumptive uses are broadly accounted as so-called water use. This misleads the public and their legislators into the false belief that there is great impending water shortage if fresh water is not obtained from the sea and saline waters.

Present and Anticipated Future Cost.—Turning now to the present (1961) and anticipated future cost of producing fresh water from the sea:

Senator Kerr's letter of July 13, 1959, to the Secretary of the Interior, to the attention of the Director of the Office of Saline Water[2] contains, in part, the following request:

> "The report should contain a brief description of desalinization processes now in use, both here and abroad, discussing engineering princi-

---

[2] Committee Print No. 8, April 1960, under the title, "Future Water Requirements of Principal Water-Using Industries," p. 16.

ples, costs per thousand gallons and per acre-foot, and environmental conditions, and actual areas within the United States suited to the application of these processes."

The Senator Kerr letter continues and asks for similar information on processes in the pilot plant stage, and the

. . ."best estimate of to what extent desalinization will be practiced by 1980, giving geographical areas and amounts of water that can be produced at various cost levels."

The reply is not quite as detailed as the request. Following is the reply by the Assistant Secretary of the Interior on October 19, 1959, (the writer has inserted in each case the equivalent cost in acre-feet):

"Just prior to World War II the cost of converting sea-water to fresh ranged upward from $4 per thousand gal ($1,304 per acre-ft). In one or more of the plants Public Law 85-833 authorized the Department to construct and operate, we expect to convert saline water to fresh for less than $1 per thousand gal $326 per acre-ft)- - - New processes, still in the laboratory stage and not advanced far enough to be considered for demonstration plant programing, give promise of lowering the cost of conversion below 50 cents per thousand gal ($163 per acre-ft)." The statement continues:

"For years the citizens of Coalinga, California, like many other communities in this country have had to haul in their drinking water supplies. In Coalinga, this water was obtained at a cost of $7 per thousand gal ($2282 per acre-ft). Earlier this year this community cut its water bill to $1.45 per thousand gal ($473 per acre-ft) and made U. S. history by becoming the first to get its drinking water supply from brackish well water."

Elsewhere from this report[2] there is extracted some additional cost data as follows:

"The cost of fresh water produced by the most efficient existing sea-water conversion plants is now approximately $1.75 per thousand gal ($570 per acre-ft). It is anticipated that the first two sea-water distillation demonstration plants will produce fresh water from the sea for $1 or less per thousand gal ($326 per acre-ft)."

Estimated costs of producing fresh water from inland saline waters were presented,[2] as shown in Table 1.

In this table, under column 2, salts are increased from 250 ppm to 500 ppm for "Projected Intermediate Size Plant," thus defeating comparability. All costs are higher, however, than the irrigator can afford to pay.

From the preceding data on water costs, it is apparent that even projected costs of obtaining fresh water from the sea or from inland saline waters are too high for agriculture. Such methods do not afford a hopeful substitute for water obtained by normal methods of surface storage and conveyance in long pipelines and aqueducts.

## LITTLE CONSUMPTIVE USE OF WATER IN HUMID EASTERN STATES

In general, the areas of the United States east of the Mississippi that have 40 in. or more of rainfall suffer little consumptive use of water, except along

TABLE 1.—ESTIMATED COSTS OF PRODUCING FRESH WATER FROM INLAND SALINE WATERS

| Original Water | Product water (ppm) | Total Initial Plant Cost | Water Costs per 1,000 gallons | | | | | Total Water Cost per acre-ft |
|---|---|---|---|---|---|---|---|---|
| | | | Plant Invest-ment | Operation and Maintenance | | | Total | |
| | | | | Membrane Replace-ment | Power and Other | Total | | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| ACTUAL FIELD EXPERIMENTAL UNIT (If operating at 25,000 gal per day production) | | | | | | | | |
| Arizona type (4,000 ppm) | 250 | $63,000 | $0.54 | $0.47 | $0.29 | $0.76 | $1.30 | $425 |
| South Dakota type (2,000 ppm) | 250 | 59,400 | .50 | .37 | .23 | .60 | 1.10 | 360 |
| PROJECTED INTERMEDIATE SIZE PLANT[a] (1.5 mgd) | | | | | | | | |
| Arizona type (4,000 ppm) | 500 | $1,650,000 | $0.23 | $0.30 | $0.27 | $0.57 | $0.80 | $260 |
| South Dakota type (2,000 ppm) | 500 | 1,310,000 | .18 | .23 | .19 | .42 | .60 | 197 |
| PROJECTED LARGE PLANT[b] (75.0 mgd) | | | | | | | | |
| Arizona type (4,000 ppm) | 250 | $46,900,000 | $0.12 | $0.07 | $0.21 | $0.28 | $0.40 | $130 |
| South Dakota type (2,000 ppm) | 250 | 33,800,000 | .09 | .05 | .11 | .16 | .25 | 82 |

[a] Based on multiples of present stack without design changes. Progress in design improvements will lower these costs. Costs slightly higher for 250 ppm.

[b] Based on possible future development of larger stacks and other technological developments.

the coast lines where sewers carry domestic and industrial wastes to the sea. The major problem is that of the treatment of these inland wastes and the treatment of raw water before re-use, as our lakes and rivers afford both the supply of water and the conveyance of wastes. Such costs are far below the foreseeable costs of producing fresh water from the sea.

## THE SEMIARID WEST

For the reasons given in the preceding section this paper will confine itself primarily to the seventeen western states lying generally westerly of the 100th meridian, and not including Alaska and Hawaii. This includes all of the desert and semiarid areas of the United States. It contains, also, along the northern Pacific coast ranges, the highest rainfall (more than 100 in.) and much of the high mountain areas of the Cascade, Sierra Nevada, and Rocky Mountain ranges where rain is generally in excess of all vegetation requirements. Such areas are the principal sources of the streams and rivers of the West.

With the exception of the cities and areas of the Pacific northwest, all other areas in the West, including nearly all cities, metropolitan areas of major population and industry, and irrigated agriculture lie principally in the regions where mean annual rainfall is low, varying from 5 in. to 20 in.

The seventeen western states occupy 61.5% of the continental United States, omitting Alaska, but average only 27% of the forty-eight states' water runoff. They include, however, in the Pacific coast states, Arizona, and Nevada, the most rapid growth in population and industry in the nation, with the single exception of Florida. Nearly all irrigated agriculture is in the West.

It is here, then, in these seventeen western states, that water supply and waste disposal problems are of truly major proportions. Here are areas of least rainfall and lowest runoff. The locations of irrigable lands, population and industry in relation to places of large rainfall and stream flow are most displaced. There are vast areas of closed basins such as the Imperial Valley of California and the Great Basin centering in Nevada and Utah. And yet, here are the nation's largest dams, reservoirs, hydroelectric plants, longest aqueducts and greatest per capita consumptive use of water. The unfamiliar will say that here is the need for fresh water from salt and saline waters.

*Major Water Projects in the West.*—One important factor in favor of the West is that it is a newly settled land. Many major customs and mistakes of the older, more populated areas of the United States and of the old world have not yet been made. The population has the vitality, spirit, and optimism of youth. Where else would one find such a bold proposal as the California Water Resources Development Bond Act for $1,750,000,000 authorized at the last session of the State Legislature, placed before the people of that state and adopted by their vote at the general election, November 8, 1960.

This project will provide 4,000,000 acre-ft per annum, three billion five hundred million gallons of water per day, to be stored and transported as far as 600 miles through a great multiple-purpose water project. Flood control, salinity control, hydroelectric power to lessen the cost of water deliveries, water for irrigation, municipal and industrial use, recreation, and fish and wildlife will all be included in this one great project. The major costs al-

located to water use are to be fully returned to the state by those benefited in the north and south San Francisco Bay area, western and southern San Joaquin Valley, San Luis Obispo and Santa Barbara counties, and Southern California from Los Angeles to San Diego. Even if fresh water from the sea were available for municipal use in southern California, its use would defeat the irrigation features of this project in the San Joaquin Valley, as irrigation would be uneconomical without municipal use to share in the aqueduct costs. In 1990, when additional water will be required, a new economic analysis can be made of costs of fresh water from the sea compared to the long aqueducts, before a second state aqueduct is constructed.

Early Municipal Water Aqueducts.—This California Water Plan is the outgrowth of such great earlier works as: 1905-13, the Los Angeles-Owens River Aqueduct, 240 miles long, extended in 1930-40 and additional 100 miles to Mono Basin; 1913-34, the Hetch-Hetchy Project for San Francisco, 167 miles; 1925-31, the Mokelumne Project for the East Bay Municipal Utility District, 100 miles; 1932-41, the Colorado River Aqueduct of The Metropolitan Water District of Southern California, 300 miles; Denver's 32-mile tunnel through the Continental Divide; and many other notable municipal water supplies.

Major Irrigation Projects.—In the field of irrigation, flood control, and hydroelectric power generation, great works of the West constructed by the United States are too numerous to mention more than a few major projects, such as: Hoover Dam and the All-American Canal; Grand Coulee Dam and the Columbia Basin; Shasta, Friant and Trinity Dams and long canals of the California Central Valley Project; the Colorado River-Big Thompson Transcontinental Divide Project; the Glen Canyon-Upper Colorado River Storage Project; and the great works of the Corps of Engineers and the Bureau of Reclamation on the Missouri River.

John Wesley Powell, who first navigated and explored the Colorado River and gave an intelligent analysis of the opportunity for man in the development of the West. He had the vision that is still being realized and unfolded in the semiarid west. It was his ideas, enthusiasm, and work that led to the founding of the United States Geological Survey and the Reclamation Service, now the United States Bureau of Reclamation, Dept. of Interior, (USBR). The lake behind Glen Canyon Dam is to be named in his honor, as Davis Dam on the Colorado River was named in honor of his nephew, Arthur Powell Davis, the early director of the Reclamation Service.

These important events and outstanding works are mentioned to indicate the imaginative and zealous manner in which the settlers of the West have been undaunted in the magnificence of their works to capture water and bring it to lands and people. The vision and deeds of the pioneers are succeeded by each generation in this fantastic West.

It is natural that the ancient and persistent idea of practical production of fresh water from the sea or from inland saline waters gains impetus in the enthusiasm of the West. In further support is the inherent faith of western people in the ability of modern science to find an answer, as it has so conspicuously done in finding the secrets of nature by research. An example of the results of the faith in, and support of, science in California is that nowhere in America is there such an aggregation of Nobel laureates in science as at

the California Institute of Technology, the University of California, and Stanford University.

## WATER AVAILABILITY AND DEMAND—WEST COMPARED TO EAST

The low average rainfall and runoff of the semiarid West in comparison to that in the humid East is shown in Table 2.

TABLE 2.—AVERAGE RUNOFF AND WATER USE IN THE UNITED STATES, 1950

| | 17 Western States | 31 Eastern States | Continental United States |
|---|---|---|---|
| Area in Acres (millions) | 1,168 | 737 | 1,905 |
| Runoff in Acre-ft (millions) | 393 | 1,057 | 1,450 |
| Runoff Depth - Inches | 4.03 | 17.23 | 9.13 |
| Irrigated Area, 1950 (millions of acres) | 22 | negligible | 22 |
| Population, 1950 (millions) | 34 | 116.7 | 150.7 |
| Population, 1960 (millions) | 44 | 134.5 | 178.5 |
| Use of Water, 1950 | | | |
| Municipal & industrial (millions acre-ft) | 10 | 80 | 90 |
| Irrigation (million acre-ft) | 90 | negligible | 90 |
| Total (million acre-ft) | 100 | 80 | 180 |

In major drainage areas of the West, runoff depths are as follows:

| Drainage Area | Runoff Depth, in inches |
|---|---|
| Great Basin | 1.0 |
| Colorado River | 1.1 |
| Missouri River - Hudson Bay | 1.9 |
| Rio Grande - Western Gulf | 3.2 |
| Arkansas-White-Red Rivers | 7.0 |
| California-Pacific | 12.0 |
| Pacific Northwest | 13.0 |
| Average of Western States | 4.03 |

These figures tend to indicate that the three Pacific coast states have a more favorable situation than others of the West, but the remaining fourteen western states may expect severe limitations on their full development, especially for irrigation. But even in these three states there is severe maldistribution of available water both in time and location. There is insufficient water for full development in the Snake River Valley, eastern Oregon, in California

generally south of Sacramento, and in the Great Basin and Colorado River drainage areas. Exhaustive studies have demonstrated that there is sufficient water in California, including its Colorado River diversions, to fully meet ultimate requirements of some 50 million population and nearly 20,000,000 irrigated acres. The necessary large storages, surface and underground, the long aqueducts, hundreds of miles long, combined with high pump lifts, indicate that the costs will be high.

Need for more conserved water in the semiarid West (with the exception of favored locations in the Northwest and northern California) is a truism that is recognized almost universally. Westerners have been conditioned to supporting large expenditures of their savings and means to construct works to assure them of adequate water supplies. They have become accustomed to look to the leading engineers to find the most economic solution to their future water supply needs. It is with this thought and background that this paper is written.

## MORE LAND THAN WATER IN THE WEST

There is far more land than can ever be irrigated from the streams and ground water of this, on the whole, water-deficient area. There are 1,500 miles of lands facing the sea. The Office of Saline Water shows that there are 320 million cubic miles of sea-water. For all practical purposes the quantity is unlimited. There are also large amounts of inland saline waters. In fact, return water from irrigated lands is contributing to these all the time, and their quantity is increasing. Almost all the nation's saline waters, with the exception of tidal coastal saline waters, and connate water at great depths, are to be found in the West. There are major problems of elevation and remoteness from sea-water and difficulty in disposal of wastes from inland extraction of fresh water from saline waters.

What are the needs for developing fresh water from the sea along the Pacific coast of the United States? In Washington and Oregon, there is a great surplus of excellent fresh water, making the use of converted sea-water permanently unnecessary. The same situation is true of the California coast north of Marin County in the San Francisco Bay area, where there are great surpluses of low-cost natural water. From there to San Diego on the Mexican border is an area where the comparable costs of water recovered from the sea and natural water transported in long aqueducts from the Sacramento-San Joaquin Delta to water-deficient areas must be, and have been, reviewed.

## CALIFORNIA VOTES $1.75 BILLION FOR INITIAL STATE WATER PROGRAM

Fortunately, California since 1947 has been engaged in a thorough study of available water supplies, present and future water requirements, and the most economic means of conveying surplus waters from the north to meet the future supplemental water needs of the balance of the state. The results of these studies are available.[3] They clearly indicate that there is sufficient natural water, including Colorado River water, to meet all future water needs within the state. Large surface and ground water storage facilities and long aqueducts will be required.

---

[3] Bulletins 1, 2 and 3, State of California Water Resources, Sacramento, Calif.

The State Water Resources Development Bond Act to authorize $1,750 million in bonds was approved at the general election, November 8, 1960. The average cost of water delivered wholesale to southern California is estimated to be $50 to $70 per acre-ft. It is planned that these works will meet the water needs of definite service areas which contract with the state from the north San Francisco Bay area to San Diego. Costs per acre-foot will be much less for shorter distances and lesser pump lifts along the aqueduct.

## FRESH WATER FROM THE SEA CANNOT NOW COMPETE WITH ADDITIONAL LONG AQUEDUCTS

While it is true that for some small isolated cases, such as that of the inland oil town of Coalinga, a water supply from local saline water may be the present answer, the high production cost of fresh water from the sea for large quantities cannot compete with the long aqueducts in California where billions of gallons of water per day (millions of acre-ft per yr) will be delivered.

The cost of the water at the Delta under the California state multiple-purpose program is only 1.075 cents per 1,000 gal ($3.50 per acre-ft). For use in the San Joaquin Valley, fresh water from the sea would have to compete with this cost. All other costs of the state's program are the costs of pumping and transmission, which would also have to be added to the conversion cost of fresh water from the sea.

Going now to the other areas of opportunity which may exist for fresh water extracted from sea-water along the Gulf Coast, Louisiana may be dismissed because of having access to large quantities of fresh water except, possibly, for a small isolated community or industry.

Along the Texas coastline the heavy rainfall of 50 in. at its eastern border becomes less to the west and south, diminishing to approximately 25 in. at Brownsville. The coastal plain of Texas differs from that of the Pacific coast. Texas has slowly rising lands continuing 100 or more miles inland from the Gulf, while all along the Pacific coast the Coast Range of mountains parallels the coast, leaving but an occasional depth of plains or low, rolling hills for large population, industry, and irrigated agriculture. The largest of these coastal plains is occupied by the Los Angeles metropolitan area.

The total runoff of Texas rivers and streams is much less than that of California, and usable lands are several times larger. Accordingly, it is conceivable that there may come a time when this region may look to the sea as an additional source of fresh water, especially for industrial use. This is, of course, dependent on delivery of such water at reasonable prices. It appears the time is far off.

## PROBLEMS OF PRODUCING FRESH WATER FROM INLAND SALINE WATERS

The inland areas of the seventeen western states have substantially all of the saline waters of the country. In general, such waters do not exist in the Columbia River and coastal drainages of the Pacific coast. They do exist throughout the Rocky Mountain states, and are extensive through the Great Plains from the Dakotas to Texas. One of the problems of continued irrigation of lands is the necessity of washing the salts downward so they will not accumulate in the soil. This drainage is frequently a serious problem.

*Disposal of Concentrated Saline Water Wastes.*—A low-cost means of producing fresh water from such inland saline waters may become of material benefit. Its extensive use, however, is subject to careful analysis of effect on streams and ground water basins. In any process for the production of fresh water from saline waters there is the necessity of getting rid of the more concentrated saline water thus created by extraction of the fresh water. In all processes there is no reduction in the total volume of salt, and in every process having a reasonable prospect of ultimate use, it is not dry salt that remains to be disposed of, but salt solutions of varying concentrations.

Presumably, recovery plants located at inland saline sinks, such as Salton Sea and the Great Salt Lake and a great many lesser surface and ground water accumulations of saline water, would have no great difficulty in disposing of the more concentrated saline water. Through the passing of time, however, the available raw saline water would become more and more concentrated, adding to the costs of recovery of fresh water.

Many states have already encountered the troublesome problem of disposing of oil field brines, which may vary from 5% to 95% of the volume of oil extracted from the oil-bearing formations. The release of such brines, which may be much more concentrated than sea-water, has severely polluted many inland streams and ground water basins. Frequently, the only remedy is to pump the waste brines from oil production back into such deep porous rocks as those from which the oil with water was initially extracted. This is by no means inexpensive, though sometimes it has other offsetting values in increased production of oil or gas.

An example[4] of the high cost of pumping waste brines into deep underground rock formations where they will not contaminate useful fresh ground water or surface streams is given by these figures: 8,948,763 barrels of brine were so pumped underground at a stated cost of $348,926.94, or $0.039 per barrel. This may not be a high cost in relation to oil at $2 to $3 per barrel, but converted to a water cost of $0.93 per 1,000 gal ($03 per acre-ft), it would be a heavy financial burden on any fresh-water plant which has to dispose of its waste brine in this manner.

## CONCLUSIONS

It is not enough to emphasize the increasing demands on our fresh-water supplies and suggest that future shortages, when they occur, may be met by salvage of fresh water from the sea and inland saline waters.

Insofar as the United States mainland is concerned, it would appear that there will be no substantial need or opportunity for such plants along the Pacific coast. A need for these plants may develop along the Texas Gulf coast when the lower-cost natural waters are fully utilized. Inland in the severnteen western states, there will develop a great need for additional sources of water, and there are substantial quantities of saline waters. This is an area where agriculture, especially irrigated agriculture, has a prominent or predominant role in the present economy of each state. As natural fresh waters become fully developed it will be difficult for agriculture on a price basis to compete

4 "Oil Well Brine Problem of the Corney Drainage System (Arkansas-Louisiana)," by Murray Stein, Twelfth Purdue Industrial Waste Conf., Purdue Univ., May 15, 1957.

with urban and industrial demands for water. Any solution offered by production of fresh water from saline waters would have to be within the ability of the irrigator to pay. It would appear that high-cost disposal of waste saline-process waters may, in many instances, offer an added problem in the use of inland saline waters as a source of fresh water.

These problems are mentioned, because, in the writer's opinion their study should parallel that of developing lower cost methods of developing fresh water from the seas and inland saline waters.

Journal of the
IRRIGATION AND DRAINAGE DIVISION
Proceedings of the American Society of Civil Engineers

# UNSTEADY FLOW OF GROUND WATER INTO DRAIN TILE

By R. H. Brooks[1]

## SYNOPSIS

A solution to the nonlinear differential equation describing unsteady flow toward equally spaced drains above a horizontal impermeable boundary is presented.

The solution is compared with field data and a published numerical solution. The solution was found to agree with the field data and the numerical solution when the drain spacing was large relative to the depth of drains.

## INTRODUCTION

On agricultural lands underlain by impermeable boundaries of zero slope or where there is little natural subsurface drainage, equally spaced drains may be installed to adequately lower the water table for crop production. The depth and spacing of the drains are two important design factors that control the lowering of the water table.

Note.—Discussion open until Nov. 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Irrigation and Drainage Division, Proceedings of the American Society of Civil Engineers, Vol. 87, No. IR 2, June, 1961.

[1] Agric. Engr., Ft. Collins, Colo.

In recent years, some approximations[2,3,4,5,6] have been obtained for the solution of drainage problems involving a falling water table. Many of the approximations are objectionable[7] because the assumptions on which they are based are not realistic and hence, the solutions may not be sufficiently accurate for design purposes. This discussion deals with improving an approximation developed by R. E. Glover,[3] F. ASCE, by eliminating an assumption in his development. Such an approximation will provide information on depth and spacing of drains as well as water table recession rates.

As reported by L. E. Dumm[3] Glover developed a solution based on the heat flow equation for the problem of the falling water table. Using the Dupuit assumption, he considered equally spaced tile drains in a homogeneous soil overlying a horizontal impermeable layer. In the development of Glover's solution an approximation was made that restricts the use of his equation to cases in which the distance from the tile to the impermeable boundary is large in relation to the drawdown of the water table. Experience has indicated[3] that when the assumptions are satisfied, satisfactory results may be obtained. In summarizing some of the recent studies on the falling water table, van Schilfgaarde[7] stated that "Glover's equation, based on the assumption of horizontal flow, appears to be more nearly correct than any other, but it is not sufficiently accurate to be used for design purposes."

The approximate solution presented herein takes into account the drawdown of the water table which implies that the distance from the tile to the impermeable boundary may be small compared to the drawdown of the water table. However, the approximate solution still involves the Dupuit or horizontal flow assumption.

## THEORY

Consider a system of equally spaced drains in a homogeneous soil overlying an impermeable boundary of zero slope as shown in Fig. 1. The equation for flow based on the Dupuit assumption and Darcy's law may be written as

$$Q = K(D + h)\,\frac{\partial h}{\partial x}, \qquad (1)$$

in which Q is the volume rate of flow in the x direction per unit length of tile, K is the hydraulic conductivity, and the other symbols are as defined in Fig. 1.

---

2 "Model Tests on a Tile-spacing Formula," by William W. Donnan, Proceedings, Soil Science Soc. of Amer., Vol. 11, 1946, p. 131.

3 "Drain Spacing Formula," by L. E. Dumm, Agricultural Engineer, Vol. 35, 1954, p. 726.

4 "A Quantitative Method for Determining Ground Water Characteristics for Drain Design," by John G. Ferris, Agricultural Engineering, Vol. 31, 1950, p. 284.

5 "Landwirtschalfliche bodenverbesserun," by J. Spottle, gen Hamb. Ing. Wiss., Part 3, Wasserbau, Vol. 7, 4th edition, Wilhelm, Englemann, Leipzig, p. 1.

6 "Depth and Spacing for Drain Laterals as Computed from Core Sample Permeability Measurements," by P. Walker, Agricultural Engineering, Vol. 33, 1952, p. 71.

7 "Approximate Solutions to Drainage Flow Problems," by Jan van Schilfgaarde, Drainage of Agricultural Lands, edited by James N. Luthin, Agronomy, Vol. 7, 1957, p. 79.

If Eq. 1 is substituted into equation of continuity, the differential equation describing flow toward the drain becomes

$$\alpha \frac{\partial^2 h}{\partial x^2} - \frac{\partial h}{\partial t} = - \frac{\alpha}{D} \left(\frac{\partial h}{\partial x}\right)^2 - \frac{\alpha}{D} h \frac{\partial^2 h}{\partial x^2} \quad \ldots\ldots\ldots \quad (2)$$

in which $\alpha$ is equal to D K/f, f is the specific yield, and t denotes time.

The boundary and initial conditions imposed on the system are given as

$$h\left(\pm \frac{L}{2}, t\right) = - \frac{H_o}{2}, \; (t \geq 0), \quad \ldots\ldots\ldots\ldots\ldots \quad (3a)$$

and

$$h(x, 0) = \frac{H_o}{2}, \; \left(- \frac{L}{2} \leq x \leq \frac{L}{2}\right). \quad \ldots\ldots\ldots\ldots \quad (3b)$$

Emile Picard's method of successive approximations[8] was suggested to the writer by Glover for solving the non-linear differential Eq. 2. Eq. 2 is not

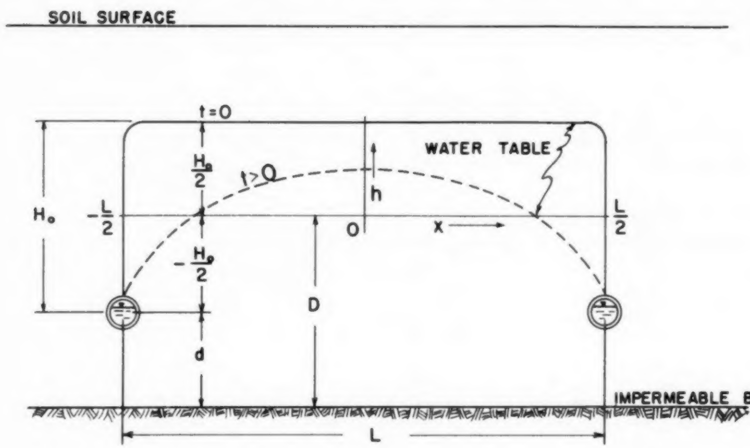



FIG. 1.—MODEL OF EQUALLY SPACED DRAINS ABOVE AN IMPERMEABLE BOUNDARY WITH ACCOMPANYING BOUNDARY AND INITIAL CONDITIONS

exactly the same type of equation considered by Picard, however, Picard's method when applied to this equation yields the same results as the method of Poincarè, Lighthill and Kuo, sometimes referred to as the PLK method.[9] The application of the PLK method to Eq. 2 has been clearly presented by

8 "Mémoire sur la théorie des equations aux derives patielles et la méthode des approximations successives," by Emile Picard, Liouville's Journal de Mathématiques, Series 4, Vol. 6, 1890, p. 145.

9 "The Poincaré-Lighthill-Kuo Method," by H. S. Tsien, Advances in Applied Mechanics, Vol. IV, Academic Press, 1956, p. 281.

sundara Raja Iyengar[10] in a discussion of a paper by Haushild and Kruse[11] wherein Eq. 2 was applied to a different set of boundary conditions.

As stated by Sundara Raja Iyengar, this method introduces a perturbation parameter into the original differential equation. In this case Eq. 2 becomes

$$\alpha \frac{\partial^2 h}{\partial x^2} - \frac{\partial h}{\partial t} + \epsilon \left[ \frac{\alpha}{D} \left(\frac{\partial h}{\partial x}\right)^2 + \frac{\alpha}{D} h \frac{\partial^2 h}{\partial x^2} \right] = 0 \quad \ldots\ldots \quad (4)$$

in which $\epsilon$ is the perturbation parameter. When $\epsilon = 0$, Eq. 4 reduces to a linear differential equation that was solved by Glover as a first approximation for the problem under consideration.

Assuming that the solution $h(x, t, \epsilon)$ may be expanded in powers of $\epsilon$, the solution becomes

$$h(x, t, \epsilon) = \sum_{i=0}^{n} \epsilon^i h_i (x, t). \quad \ldots\ldots\ldots\ldots\ldots \quad (5)$$

By substituting Eq. 5 into Eq. 4 and equating the coefficients of powers of $\epsilon$ to zero, a set of differential equations is obtained, the solutions of which provide the necessary terms in Eq. 5. This set of differential equations is given as

$$\left.\begin{array}{l}
\alpha \dfrac{\partial^2 h_1}{\partial x^2} - \dfrac{\partial h_1}{\partial t} + \dfrac{\alpha}{D} \left(\dfrac{\partial h_o}{\partial x}\right)^2 + \dfrac{\alpha}{D} h_o \dfrac{\partial^2 h_o}{\partial x^2} = 0 \\[2ex]
\alpha \dfrac{\partial^2 h_2}{\partial x^2} - \dfrac{\partial h_2}{\partial t} + \dfrac{\alpha}{D} \left(\dfrac{\partial h_1}{\partial x}\right)^2 + \dfrac{\alpha}{D} h_1 \dfrac{\partial^2 h_1}{\partial x^2} = 0 \\[2ex]
\quad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\[2ex]
\alpha \dfrac{\partial^2 h_n}{\partial x^2} - \dfrac{\partial h_n}{\partial t} + \dfrac{\alpha}{D} \left(\dfrac{\partial h_{n-1}}{\partial x}\right)^2 + \dfrac{\alpha}{D} h_{n-1} \dfrac{\partial^2 h_{n-1}}{\partial x^2} = 0
\end{array}\right\} \quad \ldots (6)$$

The first team in Eq. 5, $h_o(x, t)$, is Glover's solution and when it is modified for the boundary conditions given by Eq. 3 becomes

$$h_o = \frac{4 H_o}{\pi} \sum_{n=1,3,5,\ldots}^{\infty} (-1)^{(n-1)/2} \frac{1}{n} \exp\left(- \frac{\alpha n^2 \pi^2 t}{L^2}\right) \cos \frac{n \pi}{L} x - \frac{H_o}{2} \quad \ldots . (7)$$

The second term in Eq. 5 was obtained from the solution of the differential equation in Eq. 6 involving $h_1$ and $h_o$. Each solution for the Eqs. 6 must satisfy

10 Discussion by K. T. Raja Iyengar Sundara of "Unsteady Flow of Ground Water into a Surface Reservoir," by William Haushild and Gordon Kruse, Proceedings, ASCE, Vol. 87, No. HY 1, January, 1961, p. 280.

11 "Unsteady Flow of Ground Water into a Surface Reservoir," by William Haushild and Gordon Kruse, Proceedings, ASCE, Vol. 86, No. HY 7, July, 1960, p. 13.

the conditions given by Eq. 3. Because of the difficulty in obtaining additional terms, the solution presented subsequently involves only the first two terms of Eq. 5. The solution to the problem is presented in terms of $h_o(x, t)$ and is written as

$$h(x, t) = \left(1 + \frac{H_o}{2 D}\right) \left(h_o + \frac{H_o}{2}\right) - \frac{\alpha}{2 D} H_o \, t \frac{\partial^2 h_o}{\partial x^2} + \frac{1}{2 D} \frac{\partial h_o}{\partial x} \int \left(h_o + \frac{H_o}{2}\right) dx$$

$$- \frac{1}{2 D} \left(h_o + \frac{H_o}{2}\right)^2 - \int_o^t \frac{\partial F}{\partial T} \left(H_o - G\right) dT - \frac{H_o}{2} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots (8)$$

in which $h_o$ is given by Eq. 7, and

$$F = \frac{8 H_o^2}{\pi^2 D} \sum_{n=1,3,5,\ldots}^{\infty} \sum_{m=1,3,5,\ldots}^{\infty} \frac{1}{m^2} \exp\left[- \frac{\left(m^2 + n^2\right) \alpha \pi^2 T}{L^2}\right] \quad \ldots \ldots (9)$$

The term, G, under the time integral is equal to $\left(h_o + \frac{H_o}{2}\right)$ that in turn is a function of $[(t - T), x]$ in which T is a time variable that takes on the range of values $t \geq T \geq 0$. The particular integral

$$\frac{1}{2 D} \frac{\partial h_o}{\partial x} \int \left(h_o + \frac{H_o}{2}\right) dx$$

did not satisfy the boundary condition, hence, the time integral in Eq. 8, that was evaluated numerically, was introduced to permit restoration of the boundary condition.

A solution similar to that presented by Haushild and Kruse is also presented here for purposes of comparing solutions. A reasonable approximation for flow toward the drain in Fig. 1 is given by

$$Q = K D \frac{\partial h_o}{\partial x} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots (10)$$

in which $h << D$. A more accurate expression for flow toward the drain is given by Eq. 1.

$$Q = K\left(D + h_1\right) \frac{\partial h_1}{\partial x} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots (11)$$

An approximate solution, $h_1$, is obtained by substituting Eq. 10 into Eq. 11 that when integrated yields

$$D h_1 + \frac{h_1^2}{2} = D h_o + c \quad \ldots \ldots \ldots \ldots \ldots (12)$$

in which c is the constant of integration. Using the boundary conditions given by Eq. 3, the constant of integration is found to be $\left(\frac{H_o}{2}\right)^2$. After substituting the

value of c into Eq. 12 and rearranging, the new approximation, $h_1$, is expressed as

$$h_1 = -D + \sqrt{D^2 + 2 D h_o \left(\frac{H_o}{2}\right)^2} \quad \ldots\ldots\ldots\ldots (13)$$

This expression also satisfies the initial condition.

## THEORETICAL RESULTS AND DISCUSSION

The solution of Eq. 8 is presented graphically in Fig. 2 for x = 0 and for various values of $H_o/D$. Glover's solutions are also shown in Fig. 2 for comparison with this theory.

The reason for the apparent inconsistency between Glover's special equation for $H_o/D = 2.0$, that places the drain on the impermeable boundary, and Eq. 8 is that Glover's solution[3] does not satisfy the same initial conditions assumed herein

The curves in Fig. 2 obtained from Eq. 8, show that the water table midway between drains does not begin to recede until a finite period of time has elapsed depending on the distance from the drain to the impermeable boundary. The range of relative distances to the impermeable boundary, $0 < H_o/D \leq 2.0$, constitutes 100% of the total drainable depth. Don Kirkham[12] showed that flow into tile drains is considerably reduced when the drain is placed on the impermeable boundary. A comparison of the curves in Fig. 2 for the drain on the impermeable boundary, $H_o/D = 2.0$, and at two-thirds of the drainable depth, $H_o/D = 1.0$, also indicates that when the drain is near the boundary the flow is considerably reduced.

A comparison of the two approximations, Eq. 8 and 13, is shown in Table 1. The agreement of the two approximations is excellent for $0.1 \leq H_o/D \leq 1.0$. However, for $H_o/D = 2.0$, the agreement is not entirely satisfactory.

In a recent paper by J. D. Isherwood[13], the general method of Don Kirkham and R. E. Gaskell[14] was used with the use of a high speed computer to solve the boundary value problem presented herein. Isherwood solved the problem as a two-dimensional system; hence, his solution should be more accurate than the one-dimensional system which is based on the Dupuit assumption. By comparing Isherwood's solution with the approximate theoretical solution presented here, it is possible to determine the validity of Eq. 8 to a reasonable degree.

Isherwood's data for the case of an impermeable boundary existing 10 ft below the soil surface is reproduced in Fig. 3, using dimensionless parameters. From this comparison it seems necessary to place a restriction upon Eq. 7 and Eq. 8 and 13. This restriction may be made by use of the ratio of the tile spacing, L, to the distance from the tile to the impermeable boundary,

---

12 "Reduction in Seepage in Soil Underdrains Resulting from Their Partial Embedment in, or Proximity to, an Impervious Substratum," by Don Kirkham, Proceedings, Soil Science Soc. of Amer., Vol. 12, 1948, p. 54.

13 "Water Table Recession in Tile-drained Land," by J. D. Isherwood, Journal of Geophysical Research, Vol. 64, 1959, p. 795.

14 "Falling Water Table in Tile and Ditch Drainage," by D. Kirkham and R. E. Gaskell, Proceedings, Soil Science Soc. of Amer., Vol. 15, 1951, p. 37.

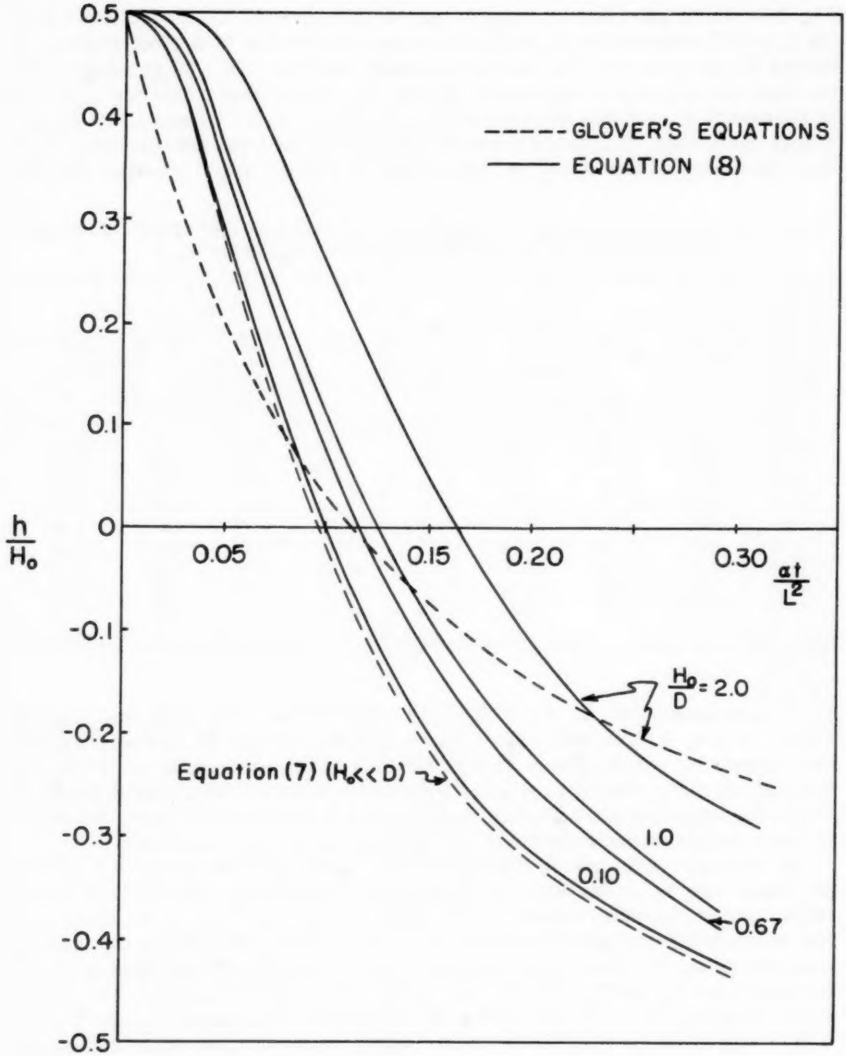



FIG. 2.—THEORETICAL CURVES OF RELATIVE WATER TABLE HEIGHT, $h/H_0$, AS A FUNCTION OF THE TIME PARAMETER $\alpha t/L^2$ FOR TILE AT VARIOUS RELATIVE DISTANCES $H_0/D$ ABOVE THE IMPERMEABLE BOUNDARY AND FOR $x = 0$

d, that is, L/d. It is reasonable to expect that for large ratios of L/d a solution to this problem based on the Dupuit assumption would closely approximate the two dimensional solution, and conversely for small ratios of L/d such a solution might be in considerable error. This fact is clearly shown in Fig. 3 in which Isherwood's solution agrees closely with the theoretical curve for $L/d \geq 25$ whereas for $L/d < 25$ there is considerable lack of agreement. It should be noted in Fig. 3(a) that Isherwood's data for L/d = 25 coincides with the theoretical curve of this paper. In Fig. 3(b), Isherwood's data for L/d = 50 is represented as points because the two curves for L/d = 25 and L/d = 50 are nearly coincident. It appears from the data available from Isherwood's paper that the theory of this paper is independent of L/d for ratios greater than 25.

TABLE 1.—A COMPARISON OF THEORETICAL SOLUTIONS OBTAINED FROM EQS. 8 AND 13 FOR x = 0 AND FOR VARIOUS VALUES OF $H_o/D$.

| Time Parameter, $\alpha t/L^2$ | $H_o/D$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 0.1 | 0.67 | 1.0 | 2.0 | 0.1 | 0.67 | 1.0 | 2.0 |
| | Relative position of the water table, $h/H_o$ | | | | | | | |
| | From Eq. 8 | | | | From Eq. 13 | | | |
| 0.02 | 0.48 | 0.49 | 0.49 | 0.50 | 0.46 | 0.48 | 0.48 | 0.49 |
| 0.03 | 0.43 | 0.46 | 0.48 | 0.49 | 0.41 | 0.44 | 0.45 | 0.46 |
| 0.05 | 0.28 | 0.33 | 0.37 | 0.46 | 0.27 | 0.32 | 0.34 | 0.38 |
| 0.07 | 0.15 | 0.22 | 0.26 | 0.38 | 0.14 | 0.21 | 0.23 | 0.30 |
| 0.10 | -0.01 | 0.07 | 0.11 | 0.25 | -0.02 | 0.06 | 0.10 | 0.19 |
| 0.16 | -0.23 | -0.16 | -0.13 | -0.01 | -0.23 | -0.17 | 0.12 | 0.01 |
| 0.20 | -0.31 | -0.25 | -0.21 | -0.11 | -0.31 | -0.26 | 0.22 | -0.08 |
| 0.25 | -0.38 | -0.33 | -0.30 | -0.22 | -0.39 | -0.34 | 0.31 | -0.16 |
| 0.30 | -0.43 | -0.40 | -0.38 | -0.28 | -0.43 | -0.41 | 0.38 | -0.24 |

A comparison of A. F. Klinge's[15] unpublished field data, and Eq. 8 is shown in Fig. 4. The theoretical curve appears as a solid line whereas the data appear as points. These data of Klinge's agree remarkably well with the theoretical cuve. These data were reported for shallow drainage systems in which the impermeable boundary was 3 ft to 4 ft below the surface. A number of these comparisons were made and similar agreement was found.

An example of the use of the theoretical curves follows. Assume a system of drains are to be installed to a depth of 6 ft below the soil surface and an impermeable boundary exists at 9 ft below the soil surface. If the position of the water table initially exists at the soil surface, compute the tile spacing that will cause the water table to recede to a depth of 3.6 ft in 3 days when the hydraulic conductivity-specific yield ratio, K/f, is 2.0 ft per hr. The scaled time variable, $\alpha\ t/L^2$, is read from the theoretical curve, $H_o/D = 1.0$ for $h/H_o = -0.1$, that is found to be 0.150, Fig. 2. Because t and $\alpha$ are known, the

15 "Physical Characteristics of Clermont Silt Loam Soil in Relation to Tile Drainage," by A. F. Klinge. Thesis presented to Purdue Univ. at Lafayette, Indiana, in 1955, in partial fulfillment of the requirements for the degree of Master of Science.

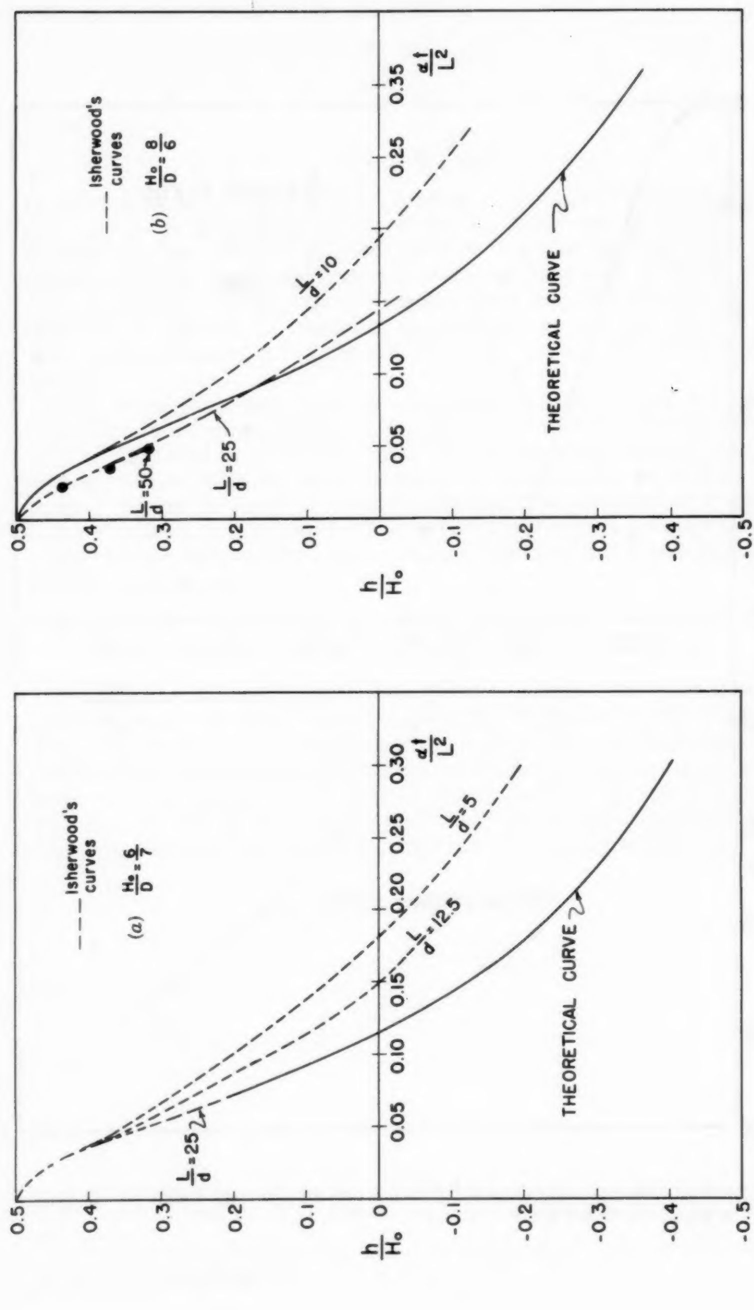



FIG. 3.—COMPARISON OF THE THEORETICAL CURVE AND THE SOLUTION OF ISHERWOOD FOR $H_O/D = 6/7$ AND 8/6 AND FOR VARIOUS VALUES OF L/D AT x = 0

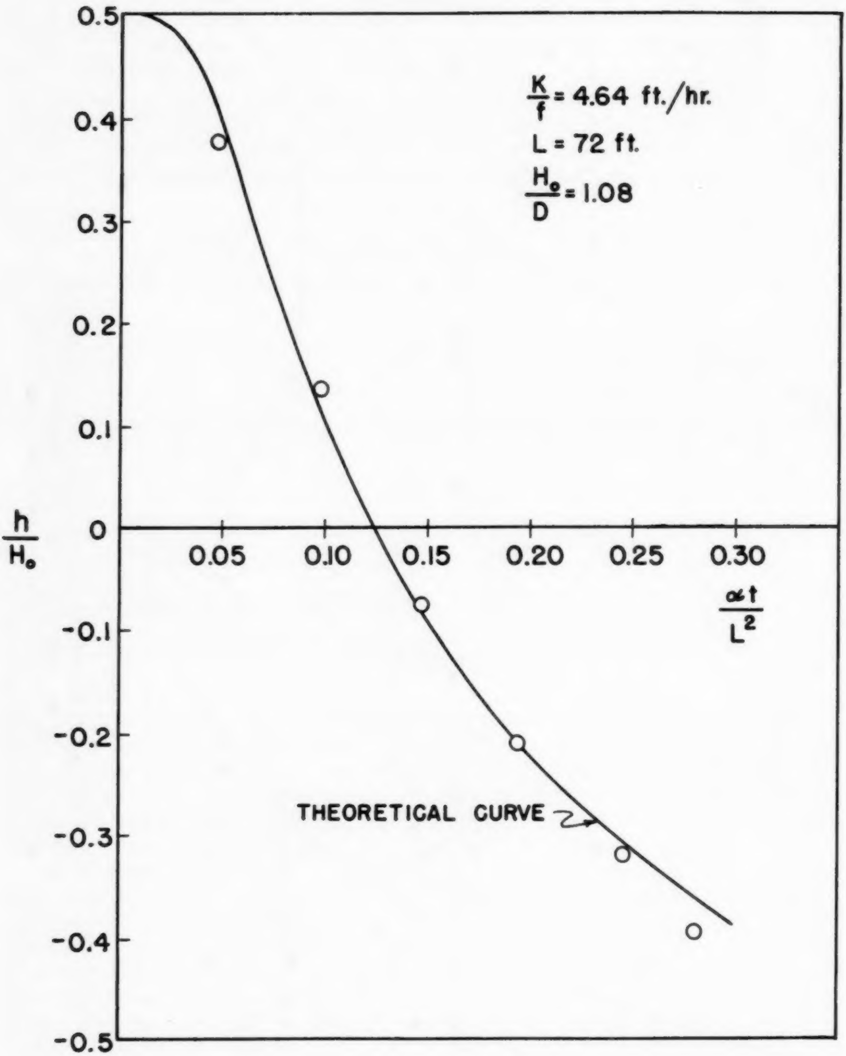



FIG. 4.—COMPARISON OF THE THEORETICAL CURVE AND THE DATA OF KLINGE FOR $H_O/D = 1.08$ AT $x = 0$

spacing, L, is computed from the relation

$$\frac{\alpha t}{L^2} = 0.150$$

or

$$L = \sqrt{\frac{\alpha t}{0.150}}$$

in which $\alpha$ is 12.0 $ft^2$ per hr. The spacing is found to be 76 ft (approximately) and L/d = 76/3 = 25.3 that is within the limits set forth using the data of Isherwood. The theoretical predictions of the tile spacings that can be obtained from Fig. 2 will be no better than the methods used to obtain characteristic hydraulic conductivity - specific yield data for the drainage system. Methods of measuring hydraulic conductivity on a large scale are needed. The recent work of R. W. Nelson[16] seems to be a step forward in this direction.

The assumption that there is a constant quantity called specific yield is indeed naive as discussed by E. C. Childs[17]. The true solution of this problem as indicated by Childs[17] "Will demand the study of the soil as a whole, both above and below the water table, as an essay in the field of water movement in a medium whose hydraulic conductivity is a function of moisture content." However, it is hoped that the approximate solution presented herein will enable the drainage engineer to more accurately design drainage installations and predict drainage costs.

## ACKNOWLEDGMENTS

This paper is a contribution from Western Soil and Water Management Research Branch, Soil and Water Conservation Research Division, Agricultural Research Service, U. S. Department of Agriculture, with the Colorado Agricultural Experiment Station cooperating. The paper resulted from a series of lectures on the mathematics of ground water movement given by R. E. Glover; consultant for the Agricultural Experiment Station, Colorado State University. His encouragement and assistance is greatfully acknowledged.

16 "In-place Measurement of Permeability in Heterogeneous Media," 1. Theory of a Proposed Method, by R. W. Nelson, Journal of Geophysical Research, Vol. 65, 1960, p. 1753.

17 "The Nonsteady State of the Water Table in Drained Land," by E. C. Childs, Journal of Geophysical Research, Vol. 65, 1960, p. 780.

Journal of the
IRRIGATION AND DRAINAGE DIVISION
Proceedings of the American Society of Civil Engineers

# WATER TABLE FLUCTUATIONS INDUCED BY IRRIGATION

By Marinus Maasland,[1] M. ASCE

## SYNOPSIS

The theory of intermittent recharge, as previously developed, is extended and analyzed. Equations are presented from which charts may be prepared for easy computation of drain spacings for irrigated agriculture. The analytical results may also be used to compute the effect of natural drainage on the height of the water table as well as anticipated changes in the water table of newly irrigated areas. Example applications of the theory to ditch and tile drainage, and natural drainage will be covered in later publications.

## INTRODUCTION

The effect of recharge on the water table depends on many factors such as characteristics of the water bearing stratum and of the materials under and overlying it; amount and distribution of the recharge; extent to which the river or drain penetrates into the water transmitting porous medium; and the depth and spacing of subsurface drains. The theory of intermittent recharge[2] has been found useful in analyzing relationships between the various factors. The mathematical treatment is somewhat complex but its results can be conveniently used for solving practical problems by preparing graphs and charts. The analysis brings together, in a rational manner, the apparently opposite

Note.—Discussion open until November 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Irrigation and Drainage Division, Proceedings of the American Society of Civil Engineers, Vol. 87, No. IR 2, June, 1961.

1 Engr., Tippetts-Abbett-McCarthy-Stratton, Engrs. and Archts., 375 Park Ave., New York, N.Y.

2 "Watertable Fluctuations Induced by Intermittent Recharge," by M. Maasland, Journal of Geophysical Research, Vol. 64, 1959, pp. 549-559; (a) p. 551; (b) p. 557; (c) p. 556.

assumptions of instantaneous recharge[3] and of continuous, constant recharge.[4,5] The analytical results may be used to determine the following:

1. Drain spacings for irrigated agriculture;
2. Applicability and limitations of the Glover formula[3] and of steady state equations for computation of drain spacings;
3. Combined effect of excess irrigation applications, rainfall, and natural drainage on the water table for various drain spacings, and groundwater and soil conditions;
4. Water table levels that ultimately will be reached in aquifers under the influence of irrigation;
5. Number of years needed for the water table to approach the new equilibrium level in newly developed irrigation projects, following increased recharge due to irrigation;
6. Effect of various amounts and methods of application of excess irrigation water on the height and fluctuation of the water table;
7. Type and distribution of recharge received by the groundwater reservoir from detailed field studies of water tables and discharges of natural streams or drainage systems; and
8. Quantity and monthly distribution of the return flow induced by deep percolation losses on irrigation projects.

The equations aid materially in evaluating recharge conditions of groundwater basins and drainage needs for irrigated areas. They also permit estimating the time when drainage construction will be necessary for newly developed irrigated areas so that funds and equipment for this purpose can be scheduled. High water tables and attendant salinity problems may thus be averted by a carefully planned program.

It is known that the first Glover formula[3] may lead to narrow spacings that are inconsistent with field experience. In applying that formula, the misleading assumption is made that it is necessary to remove from the root zone, or from the soil layers below it, the total amount of deep percolation loss from each irrigation prior to applying the next irrigation. Maasland indicates[2] that the Glover formula may be used to compute the decline of the water table following a single instantaneous recharge. It does not account for either the accumulative effect of successive recharges or the depletion of the groundwater between or following a number of recharges. However, groundwater storage during the irrigation season and its subsequent depletion during the non-irrigation season have a significant effect on the water table level. By taking these factors into account, it can be shown that permissible drain spacings may become much wider than those computed from the Glover formula. It will be shown elsewhere that the various methods of determining drain spacings and the spacing formulas derived, recommended, or used by William W. Donnan,[6] F. ASCE, et al., L. D. Dumm,[3] J. D. Isherwood,[7] S. B. Hoog-

---

3 "Drain Spacing Formula," by L. D. Dumm, Agricultural Engineering, Vol. 35, 1954, pp. 726-730.

4 "Bijdragen tot de kennis van enige natuurkundige grootheden van de grond," by S. B. Hooghoudt, Verslag, Landbouwk, Onderzoek, Vol. 46, 1940, pp. 515-701.

5 "Seepage of Steady Rainfall Through Soil into Drains," by D. Kirkham, Transactions, Amer. Geophysical Union, Vol. 39, 1958, pp. 892-908 (a) p. 900, Eqs. 84-87.

6 "Drainage Investigations in Imperial Valley, California, 1941-1951 (A 10-yr summary)," by W. W. Donnan, G. B. Bradshaw, and H. F. Blaney, U. S. Dept. of Agric., Soil Conservation Service, Tech. Pub. 120, 1954.

7 "Watertable Recession in Tile-Drained Land," by J. D. Isherwood, Journal of Geophysical Research, Vol. 64, 1959, pp. 795-804.

houdt,[4] Maasland,[8] Maasland and H. C. Haskew,[9] J. D. Van Schilfgaarde,[10,11] et al., and D. Kirkham[5] often require adjustment of the recharge coefficient if they are to be used to compute drain spacings for irrigated agriculture. Large errors may result if the recharge conditions are not properly considered.

The equations for intermittent recharge are derived on the assumption that the drains completely penetrate the permeable stratum. If the drains penetrate only part of the permeable stratum, Eqs. 29, 70, 77, 78, 82, and 83 of Hooghoudt[4] may be used to account for flow convergence near the drain.[8a] The work of Kirkham[5a] may be similarly used and leads to approximately the same results as do Hooghoudt's equations. As a matter of interest, it is observed that Hooghoudt's Eq. 29 and Kirkham's Eq. 87 are identical. This identity follows immediately from Wallis' infinite product for $\pi/2$. Correction for flow convergence may be done by a graphical method as previously presented by the writer.[8]

It is emphasized that this paper is restricted to an examination and extension of the theory of intermittent recharge. Applications will follow in subsequent works in which appropriate graphs and charts will be presented. The charts, which relate various dimensionless parameters, allow rapid evaluation of a variety of groundwater conditions.

*Notation.*—The letter symbols adopted for use in this paper are defined where they first appear, in the illustrations or in the text, and are arranged alphabetically, for convenience of reference, in the Appendix.

## THEORY

*General.*—A review and elaboration of the theory follows to facilitate exploration of the practical implications of the analytical results. Frequent reference will be necessary to the writer's previous work[2] and to equations in it. For reasons of convenience and to prevent repetition, the previous paper[2] will be referred to as (MM). Its equations will be indicated by their number preceded by the letter M. For example, Eq. 33 of (MM) is referred to as Eq. M.33. The notation used in the present paper is generally the same as that used in (MM). A few small changes have been made which will be indicated.

In the previous paper,[2] two related types of intermittent recharge were considered separately; intermittent constant recharge (recharge applied intermittently at a constant rate), and intermittent instantaneous recharge (recharge applied instantaneously at regular intervals). The effect of intermittent recharge on the water table and on the rate of discharge of the drain ('base flow' in the case of natural streams, or 'return flow'; a term used to denote the discharge induced by deep percolation losses from excess irrigation applications) may each be studied by considering simultaneously two functions

8 "The Relationship Between Permeability and the Discharge, Depth, and Spacing of Tile Drains," by M. Maasland, Groundwater and Drainage Series, Water Constr. and Irrig. Committee, N. S. W., Australia, Bulletin 1, 1956, p. 35; (A) pp. 21-33.

9 "The Auger Hole Method of Measuring the Hydraulic Conductivity of Soil and its Application to Tile Drainage Problems," by M. Maasland, Third Congress, Internatl. Committee on Irrig. and Drainage, Question 8:8.69-8.114, 1957.

10 "Physical and Mathematical Theories of Tile and Ditch Drainage and Their Usefulness in Design," by J. Van Schilfgaarde, D. Kirkham, and R. K. Frevert, Iowa State College Agric. Experiment Sta., Bulletin 436, 1956, pp. 667-704.

11 "Drainage of Agricultural Lands, Agron," by J. N. Luthin, Amer. Soc. Agron., Monograph No. 7, Madison, Wis., 1957, p. 620.

such as a periodic one and a transient one. The height of the water table or the rate of discharge at any time is found by subtracting the transient from the periodic function. The transient is identical to the periodic function at $t = 0$, t being the time since the beginning of the first recharge. After sufficient time has elapsed the transient becomes negligibly small. The water table and the discharge are then completely defined by the periodic function.

The factor $\mu$ of (MM) was not given a subscript n because it occurred so frequently in the analysis. The subscript was not needed in the derivations as long as the dependence of $\mu$ on the number n of the term in the series was realized. The subscript will now be used consistently because it is essential in the application of the theory.

The numerical value of the dimensionless parameter

$$\mu_o T = \left(\frac{KD}{\epsilon}\right)\left(\frac{\pi}{L}\right)^2 T \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

effectively determines the fluctuations and changes of the water table and drain discharge fluctuations in response to intermittent recharge. This parameter is therefore named the 'aquifer response coefficient.' For low values of $\mu_o T$ (less than 0.2), the amplitude of fluctuation as caused by intermittent recharge (of rate $p = q/t_1$) is small in comparison to the average water table level or discharge. In that case, the effect of intermittent recharge is, for practical purposes, equivalent to that of a continuous constant recharge at a lower rate $p_1 = q/T$ even if the interval of recharge $t_1$ is considerably less than T. The simple steady state Eq. M.3 and the nonstationary Eq. M.7 for continuous constant recharge may then be applied to problems actually involving intermittent recharge. The amplitude of fluctuation can be estimated from $q/\epsilon$ or a fraction thereof. It may be shown that, in the limit for $\mu_o T \rightarrow 0$, Eq. M.23 with $p = q/t_1$ is identical to Eq. M.3 with $p_1 = q/T$. The phenomenon referred to is independent of the interval of recharge $t_1$. Proof will be given later. For high values of $\mu_o T$, the problem of intermittent recharge degenerates effectively into consideration of disconnected single recharges, that is, the water table lowers to zero level before a new recharge is applied.

The general equation for the water table at any time and for any x is

$$h_x = h_{p,x} - h_{t,x} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which $h_x$ represents the height of the water table measured from the line connecting the water levels in the drains. Note that $h_x$, as defined here, equals $(z - h_1)$ of (MM). Eq. 2 implies the assumption that Eq. M.2 is applicable to all problems considered. Since solutions of Eq. M.2 are additive, $h_x$ may also be taken to be an added height to the water table prevailing before the beginning of the newly introduced intermittent recharge.

If H (Fig. 1) is small,[2] Eqs. M.2 and M.3 may be inapplicable, so that Eqs. M.5 and M.6 must be used instead. This requires that all equations of (MM) be transformed as indicated in (MM).[2a] In addition, the analytical procedures developed for intermittent recharge may then not lead to sufficiently accurate results if D increases significantly as the water table rises. Variation of D causes $\mu$ to change, which invalidates Eqs. M.17, M.18 and M.26, because Eqs. M.14 and M.15 and S of Eq. M.25 no longer represent geometric series.[2a] Recourse must then be taken to a tedious tabular procedure involving the use of the (properly transformed) Eqs. M.13, M.14, M.15, and M.25. A tabular procedure for dealing with these problems has been developed by W. N. Tapp

and Dumm. Another tabular procedure is used by R. E. Glover, F. ASCE, and M. W. Bittinger.[12a] To keep this paper within bounds, the methods referred to will not be examined herein. The principle of superposition, used in both the tabular and analytical methods, is not a new one. It has been applied to various ground water problems by J. Boussinesq,[13] J. H. Edelman,[14] Jacob, Hooghoudt,[4] Kirkham,[11] D. A. Kraijenhoff,[15] P. W. Werner,[17] and others, and is widely used in operational mathematics.[18,19]

The symbols $h_p$ and $h_{pi}$, and $h_t$ and $h_{ti}$ were used in (MM) to define the water table level for any x. These symbols will now be used exclusively in reference to the level midway between the drains. The equivalent symbols for the water table level for any x are given the subscript x, that is, $h_{p,x}$ and $h_{pi,x}$, and $h_{t,x}$ and $h_{ti,x}$. Thus the symbols are given a subscript for the general problem, while, for the special case in which $x = 1/2L$, the symbol h

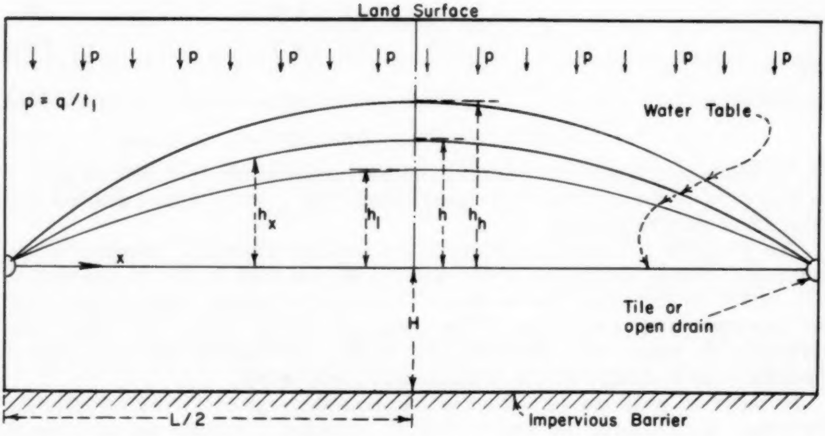



FIG. 1.—GROUNDWATER FLOW INTO DRAIN OVER A HORIZONTAL IMPERVIOUS BARRIER

12 "Transient Ground Water Hydraulics," by R. E. Glover and M. W. Bittinger, Colorado State Univ., Fort Collins, Colo., No. CER59REG16, 1956, p. 57; (a) pp. 43-44.

13 "Note sur le Mouvement des Eaux Souteraines. Essai sur la Theorie des Eaux Courantes," by J. Boussinesq, Memoires, l'Academie des Sciences, Vol. 23, France. 1877, pp. 242-281.

14 "Over de berekening van grondwaterstroomingen (Calculation of the Flow of Groundwater), thesis presented to Delft in 1947, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

15 "A Study of Non-Steady Groundwater Flow Special Reference to a Reservoir-Coefficient," by D. A. Van De Leur Kraijenhoff, De Ingenieur, Utrecht, Vol. 70, 1958, pp. 87-94.

16 "On Non-Artesian Groundwater Flow," by P. W. Werner, Geofis, Pura Appl., Vol. 25, 1953, pp. 37-43.

17 "Some Problems in Non-Artesian Groundwater Flow," by P. W. Werner, Transactions, Amer. Geophysical Union, Vol. 38, 1957, pp. 511-518.

18 "Modern Operational Mathematics in Engineering," by R. V. Churchill, McGraw-Hill Book Co., New York, 1948, p. 306.

19 "Conduction of Heat in Solids," by H. S. Carslaw and J. C. Jaeger, Oxford Univ. Press, 1959, p. 510.

remains without a subscript identifying the horizontal distance from the drain. There is no need to further define the rate of discharge of the drain Q, since the (identical) discharges at x = 0 or L are the only ones considered.

The following transformations are introduced:

$$\gamma = \frac{\tau}{T} \quad \text{(3a)}$$

$$\zeta = \frac{t_1}{T} \quad \text{(3b)}$$

$$\delta = \frac{(\tau - t_1)}{T} = \gamma - \zeta \quad \text{(3c)}$$

and

$$\xi = \frac{t}{T} \quad \text{(3d)}$$

and from Eq. M.21:

$$\alpha(\gamma) \begin{cases} = 1 & 0 \leq \gamma < \zeta \quad \text{(4a)} \\ = 0 & \zeta \leq \gamma < 1 \quad \text{(4b)} \end{cases}$$

and

$$\beta(\gamma) \begin{cases} = 0 & 0 \leq \gamma < \zeta \quad \text{(4c)} \\ = 1 & \zeta \leq \gamma < 1 \quad \text{(4d)} \end{cases}$$

The symbols $\gamma$, $\delta$, $\zeta$ and $\xi$ are dimensionless variables and their unit is equivalent in time to the interval T; $\gamma$ varies from 0 to 1, while $\xi$ may assume any value from zero to infinite.

The periodic function for intermittent constant recharge consists of two equations. These equations, which were combined into a single expression (Eq. M.23) to facilitate the analysis, will now be stated separately for the two intervals and their limits. The equations for the rate of discharge, the derivation of which was indicated (Eq. M.39), are also stated. Intermittent instantaneous recharge will be examined in some detail.

In order to evaluate the effect of drainage systems on the water table for irrigation agriculture, the theory will be extended to include the problem of 'double periodicity.' It is convenient to divide each year into two periods such as the irrigation season and the nonirrigation season. For the irrigation season, it will be assumed that the ground water is recharged at regular intervals with every irrigation or with each alternate irrigation. During the nonirrigation season, there is presumably no recharge. The problem of double periodicity requires only a slight extension of the theory developed for 'single periodic' intermittent recharge.

The assumption that there is no recharge during the nonirrigation season is a restrictive one and makes the theory inapplicable for certain areas. It can be applied only if the recharge during the nonirrigation season is small as compared to the recharge during the irrigation season. It is, of course, possible to extend the theory to include recharge during the nonirrigation season. However this would further complicate the equations and the writer plans to show, in subsequent publications, that such complications are not usually necessary. If a highly irregular recharge distribution must be considered, it is desirable to use a tabular rather than an analytic procedure. A tabular procedure for computing tile drain spacings has been described by Dumm.[3]

*The Water Table for Intermittent Constant Recharge.*—Eq. M.23 is the general expression for the periodic part of the equation for the water table. This expression must be evaluated for particular values of $\tau$ (in which $0 \leq \tau \leq T$) and is applicable to any interval $(r - 1)\ T \leq t \leq r\ T$. If the variables $\gamma$, $\delta$, and $\zeta$, defined by Eqs. 3, are entered in Eq. M.23, it becomes

$$h_{p,x} = \left(\frac{P_c L^2}{8}\right)\left\{\left[4\left(\frac{x}{L}\right)\left(\frac{1 - x}{L}\right)\right] \alpha(\gamma) - \left(\frac{32}{\pi^3}\right) \sum_{n=o}^{\infty} (2n + 1)^{-3} \sin (2n + 1)\ \pi\left(\frac{x}{L}\right)\left[\left(1 - a_n\right) \exp - \mu_n T \gamma - \beta(\gamma) \exp - \mu_n T \delta\right]\right\} \quad (5)$$

in which

$$a_n = \frac{\left(\exp \mu_n T \zeta - 1\right)}{\left(\exp \mu_n T - 1\right)} \quad (6)$$

Eq. 5 may be written as

$$h_{p,x} = \left(\frac{P_c L^2}{8}\right) w_x \quad (7)$$

in which $w_x$ is that part of Eq. 5 between braces. The term $w_x$ is determined entirely by the dimensionless parameters $\mu_o$ T, $\frac{x}{L}$, $\gamma$, and $\zeta$. The subscript x of $w_x$ is used to indicate that this expression is valid for $0 \leq x \leq L$.

In the case of intermittent constant recharge, each period T consists of two intervals such as one during which recharge takes place (at a rate $p = q/t_1$) and the other without recharge. Due to Eq. 4, the periodic function consists of a separate equation for each of these intervals. These equations do not have the same form but are identical at the limits of the intervals, that is, for $\gamma = 0$ and 1, and for $\gamma = \zeta$. It has been emphasized (MM) that Eq. 5 is single-valued for all values of $\gamma$.

For the special case that $x = 1/2L$, the equations for the intervals of $\gamma$ and their limits will be stated separately. For $\gamma = 0$ or 1, $h_p$ is minimum and is defined by the symbol $h_{p1}$. Then (M.36)

$$h_{p1} = \left(\frac{P_c L^2}{8}\right)\left(\frac{32}{\pi^3}\right) \sum_{n=o}^{\infty} (2n + 1)^{-3} (-1)^n a_n \quad (8a)$$

or

$$h_{p1} = \left(\frac{P_c L^2}{8}\right) w_1 \quad (8b)$$

Eq. 8(a) follows immediately from Eq. 9(a) by putting $\gamma = 0$, or from Eq. 10(a) by putting $\gamma = 1$ (MM). For $0 \leq \gamma \leq \zeta$, Eqs. 4 and 5 yield

$$h_p = \left(\frac{P_c L^2}{8}\right)\left\{1 - \left(\frac{32}{\pi^3}\right) \sum_{n=o}^{\infty} (2n + 1)^{-3} (-1)^n \left(1 - a_n\right) \exp - \mu_n T \gamma\right\} \quad (9a)$$

or

$$h_p = \left(\frac{P_c L^2}{8}\right) w_2 \quad (9b)$$

The maximum water table level, defined as $h_{ph}$, occurs at $\gamma = \xi$ and is given by Eq. 9(a) or Eq. M.37. For $\xi \leq \gamma \leq 1$, it follows that

$$h_p = \left(\frac{P_c\ L^2}{8}\right)\left(\frac{32}{\pi^3}\right)\sum_{n=o}^{\infty} (2n+1)^{-3} (-1)^n \left[\exp - \mu_n\ T\ \delta - \left(1 - a_n\right) \exp - \mu_n\ T\ \gamma\right] \ldots (10a)$$

or

$$h_p = \left(\frac{P_c\ L^2}{8}\right) w_3 \ldots\ldots\ldots\ldots\ldots (10b)$$

Although $h_{pl}$ and $h_{ph}$ may, in principle, be computed from Eq. 10(a), they are more conveniently obtained from Eqs. 8(a), 6, and 9(a), respectively, because the latter equations converge more rapidly than does Eq. 10(a) for the particular values of $\gamma$ ( $= \xi$ and 1).

If $x \neq 1/2L$, the factors w of Eqs. 8(b), 9(b), and 10(b) may be written as $w_{1,x}$, $w_{2,x}$, and $w_{3,x}$, respectively, to conform with the notation previously introduced for $h_{p,x}$ and $h_{t,x}$. The water table level for $x \neq 1/2L$ is obtained by substituting $\sin\ (2n + 1)\ \pi\ (x/L)$ for $(-1)^n$ in these equations.

The equation for the transient (Eq. M.24) is rewritten as

$$h_{t,x} = \left(\frac{P_c\ L^2}{8}\right)\left(\frac{32}{\pi^3}\right)\sum_{n=o}^{\infty} (2n+1)^{-3} \sin (2n+1)\ \pi\left(\frac{x}{L}\right) a_n \exp - \mu_n\ T\ \xi \ \ldots (11)$$

in which $T\ \xi = t$, the time since the beginning of the first recharge; $\xi$ may assume any value from zero to infinite.

*The Rate of Discharge for Intermittent Constant Recharge.*—The rate of discharge per unit length of drain (Q) may be computed from Eq. M.39 using Eq. 2, that is,

$$Q = K\ D\left(\frac{\partial h_{p,x}}{\partial x} - \frac{\partial h_{t,x}}{\partial x}\right)\Bigg|_{x\,=\,o} \ldots\ldots\ldots (12)$$

The discharge Q is separated into its periodic and transient components

$$Q = Q_p - Q_t \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

in which $Q_p$ denotes the rate of discharge computed from the periodic function, and $Q_t$ is the rate of discharge computed from the transient function. It is noted that Q as defined herein represents the inflow from one side of the drain only.

Discharges, as computed from the periodic part, will be considered first. From Eq. 5 and the first part of Eq. 12, the following expression is obtained for $Q_p$ at x = 0 or L:

$$Q_p = \left(\frac{p\ L}{2}\right)\left\{\propto(\gamma) - \left(\frac{8}{\pi^2}\right)\sum_{n=o}^{\infty} (2n + 1)^{-2}\left[\left(1 - a_n\right) \exp - \mu_n\ T\ \gamma - \beta(\gamma) \exp - \mu_n\ T\ \delta\right]\right\} \ldots\ldots\ldots (14a)$$

or

$$Q_p = \left(\frac{p\ L}{2}\right) d \ldots\ldots\ldots\ldots\ldots (14b)$$

in which d is that part of Eq. 14(a) between braces; d is determined entirely by the dimensionless parameters $\mu_o$ T, $\gamma$, and $\xi$.

The minimum rate of discharge, defined as $Q_{p,min}$, occurs at $\gamma = 0$ or 1, and, from Eqs. 4 and 14(a), is given by

$$Q_{p,min} = \left(\frac{p\,L}{2}\right)\left(\frac{8}{\pi^2}\right)\sum_{n=o}^{\infty} (2n+1)^{-2}\, a_n \quad \ldots\ldots\ldots (15a)$$

or

$$Q_p = \left(\frac{p\,L}{2}\right) d_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (15b)$$

For $0 \leq \gamma \leq \zeta$, Eqs. 4 and 14(a) yield

$$Q_p = \left(\frac{p\,L}{2}\right)\left\{1 - \left(\frac{8}{\pi^2}\right)\sum_{n=o}^{\infty} (2n+1)^{-2}\left(1 - a_n\right)\exp - \mu_n\, T\, \gamma\right\} \ldots (16a)$$

or

$$Q_p = \left(\frac{p\,L}{2}\right) d_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (16b)$$

The maximum discharge, defined as $Q_{p,max}$, occurs at $\gamma = \zeta$ and is computed from Eq. 16(a). For $\zeta \leq \gamma \leq 1$, it follows that

$$Q_p = \left(\frac{p\,L}{2}\right)\left(\frac{8}{\pi^2}\right)\sum_{n=o}^{\infty} (2\,n+1)^{-2}\left[\exp - \mu_n\, T\, \delta - \left(1 - a_n\right)\exp \mu_n\, T\, \gamma\right] \ldots (17a)$$

or

$$Q_p = \left(\frac{p\,L}{2}\right) d_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots (17b)$$

The rate of discharge as computed from the transient (Q) is obtained by evaluating the second part of Eq. 12 for x = 0, using Eq. 11. The result is

$$Q_t = \left(\frac{p\,L}{2}\right)\left(\frac{8}{\pi^2}\right)\sum_{n=o}^{\infty} (2\,n+1)^{-2}\, a_n \exp - \mu_n\, T\, \xi \quad \ldots\ldots (18)$$

Eq. 18 is monotonically decreasing and becomes negligibly small after sufficient time has elapsed. When this state of dynamic equilibrium is reached, the rate of discharge is determined entirely by the periodic function, and the total discharge $(D_T)$ per period T will be equal to the total recharge per period (Eq. 29).

The variation in the rate of discharge decreases as $\mu_o$ T decreases, and it may be shown, as is done for the water table in a later section, that steady state flow would obtain in the limit for $\mu_o\, T \to 0$, after dynamic equilibrium has been reached. It also follows that, for low values of $\mu_o$ T, many periods T are needed to approach the rate of discharge as computed from the periodic function, in other words, to reach dynamic equilibrium.

*Intermittent Instantaneous Recharge.*—The equations for the water table level for instantaneous recharge need not be repeated herein, as they have been adequately covered in the previous paper (MM). The water table level at any time is given by Eq. M.28, while Eq. M.29 is the periodic function and Eq. M.30 is the transient. The minimum water table level for any x and for dynamic equilibrium $\left(h_{pim,x}\right)$ is given by Eq. M.31, and the minimum equilibrium level midway between the drains $\left(h_{pi1}\right)$ by Eq. M.32. Maximum levels are obtained by adding $\left(\frac{q}{\epsilon}\right)$ to Eq. M.31 or Eq. M.32. The symbols $h_{pi,x}$ and $h_{ti,x}$ as used herein refer to the water table level for any x, and $h_{pi}$ and $h_{ti}$ are used to define the level midway between the drains.

The rate of discharge per unit length of drain $\left(Q_i\right)$ may be obtained from Eqs. M.28 and M.39

$$Q_i = K D \left(\frac{\partial h_{pi,x}}{\partial x} - \frac{\partial h_{ti,x}}{\partial x}\right)\Bigg|_{x = 0} \text{ . . . . . . . . . . .(19)}$$

The discharge $Q_i$ is separated into its periodic and transient components

$$Q_i = Q_{pi} - Q_{ti} \text{ . . . . . . . . . . . . . . . . . . . .(20)}$$

in which $Q_{pi}$ is the rate of discharge computed from the periodic function, and $Q_{ti}$ denotes the rate of discharge computed from the transient function. It is noted that $Q_i$ represents the inflow from one side of the drain only.

From Eq. M.29 and the first part of Eq. 19 it follows that

$$Q_{pi} = KD\,(q/\epsilon)\,(4/L) \sum_{n=o}^{\infty} \left(1 + b_n\right) \exp - \mu_n\, T\, \gamma \text{ . . . . . . .(21)}$$

and from Eq. M.30 and the second part of Eq. 19

$$Q_{ti} = K\,D\left(\frac{q}{\epsilon}\right)\left(\frac{4}{L}\right) \sum_{n=o}^{\infty} b_n \exp - \mu_n\, T\, \xi \text{ . . . . . . . . . .(22)}$$

in which

$$b_n = \frac{1}{\left(\exp\ \mu_n\, T - 1\right)} \text{ . . . . . . . . . . . . . . . .(23)}$$

It is remembered [(MM) and Eq. 3] that $t = (r - 1)\,T + \tau$, or $\xi = (r - 1) + \gamma$.

Eq. 21 does not converge for $\tau = 0$. Physically, this is to be expected because instantaneous recharge implies the assumption of recharge at infinite rate. The rate of discharge is finite for $0 \leq \tau \leq T$ and may be computed from Eq. 21. However, the rate of convergence of the series in Eqs. 21 and 22 is slow, which makes the use of these equations unattractive. It is often more convenient to compute the volume of the water stored below the water table and to determine the rate of discharge from the decline in the storage. The needed computations may be shortened by computing for various times the amount of ground water stored below the water table and plotting the 'mass-curve.[20] The rate of discharge at any time is then obtained by drawing a tangent to the mass-curve. This method may also be useful for intermittent constant recharge if Eqs. 14(a) and 18 converge slowly. A description and application of the principle of this method may be found in the work of Glover. and others.[21a] The procedure derives its merit from the fact that integration of the series in Eqs. M.29 and M.30 improves their rate of convergence, while differentiation has the opposite effect. Kraijenhoff considers,[15] in detail, the effect of surface-applied recharge on the ground water storage and its depletion for single recharge applications.

The storage may be determined as follows. The water table level for any x is given by Eq. M.28, and the volume of water stored below the water table at any time is obtained by integrating Eq. M.28 with respect to x from 0 to L

20 "Water Supply Engineering," by H. E. Babbitt and J. J. Doland, McGraw-Hill Book Co., New York, 1955, p. 608.

21 "Cooling of Concrete Dams," by R. E. Glover, Boulder Canyon Project, Final Reports, U. S. Bur. of Reclamation, Bulletin 3, 1949, p. 236; (a) p. 34.

and multiplying the result by $\epsilon$. Thus

$$S_i = \epsilon \int_0^L h_{pi,x}\, dx - \epsilon \int_0^L h_{ti,x}\, dx \qquad (24)$$

in which $S_i$ denotes the volume of ground water stored below the water table per unit width of aquifer and between two drains spaced at a distance L.

The volume $S_i$ will be separated into $S_{pi}$ and $S_{ti}$, the storage computed from the periodic function and the transient, respectively. It is found from the first term of Eq. 24 that

$$S_{pi} = q\, L\left(\frac{8}{\pi^2}\right) \sum_{n=o}^{\infty} (2n+1)^{-2} \left(1 + b_n\right) \exp - \mu_n\, T\, \gamma \qquad (25)$$

and from the second term

$$S_{ti} = qL \left(\frac{8}{\pi^2}\right) \sum_{n=o}^{\infty} (2n+1)^{-2}\, b_n \exp - \mu_n\, T\, \xi \qquad (26)$$

The ratio $\frac{S_i}{r\,q\,L}$ represents the fraction of the original volume of drainable water retained in the aquifer. The portion which has been discharged through the drain is equal to $\left(\frac{1 - S_i}{r\,q\,L}\right)$. The mean water table is given by the expression $\frac{S_i}{\epsilon\, L}$.

Eq. 26 is monotonically decreasing and becomes negligibly small after sufficient time has elapsed. After dynamic equilibrium has been reached, the total discharge $(D_T)$ per period T will be equal to the recharge per period, that is, 1/2q L. The factor 1/2 is used herein because the inflow from only one side of the drain is considered (Eq. M.39). This statement, obvious from a physical point of view, may be proven as follows. Integrate Eq. 21 with respect to $\tau$ (in which $\tau = \gamma$ T) from 0 to T, or

$$D_T = K\, D \left(\frac{q}{\epsilon}\right)\left(\frac{4}{L}\right) \sum_{n=o}^{\infty} \left(1 + b_n\right) \int_0^T \exp - \mu_n\, \tau\, d\tau \qquad (27)$$

Using the relationship

$$\left(\frac{8}{\pi^2}\right) \sum_{n=o}^{\infty} (2n+1)^{-2} = 1 \qquad (28)$$

it follows immediately that

$$D_T = \frac{1}{2}\, q\, L \qquad (29)$$

The same result (Eq. 29) may be obtained from Eq. 14(a) for intermittent constant recharge.

*The Factor $a_n$.*—The factor $a_n$ (Eq. 5) occurs in all problems of intermittent constant recharge. The expression for $a_n$ may be simplified for both high and low values of $\mu_o$ T. The simplified expressions may be used if the results as computed from them are within the range of accuracy required.

Since exp $\mu_n$ T $\xi$ and exp $\mu_n$ T become large for large values of $\mu_n$ T $\xi$, the number one in the numerator and in the denominator of Eq. 6 may both be

deleted, so that

$$a_n \approx \exp - \mu_n T\,(1 - \xi), \text{ for } \mu_n T \xi > N \ldots\ldots\ldots\ldots (30)$$

in which N is a numerical value of predetermined magnitude.

For small values of $\mu_n T$, $a_n \approx \theta = \frac{t_1}{T}$, and will exactly equal $\frac{t_1}{T}$ if $\mu_n T \rightarrow 0$, that is

$$\mu_n T \rightarrow 0 \lim a_n = \theta = \frac{t_1}{T} \ldots\ldots\ldots\ldots (31)$$

Fig. 5 of (MM) shows curves of $a_n$ versus $\mu_n$ T for each of the constant values $\xi$ = 0.2, 0.4, 0.6, and 0.8. Inspection of this figure shows that, for all four curves, $a_n \approx t_1/T$ for $\mu_n$ T = 0.01, the lowest value for which $a_n$ has been plotted.

*The Water Table for Low Values of* $\mu_o$ *T.*—It may be shown that the water table would not fluctuate in response to intermittent recharge if $\mu_o T \rightarrow 0$. This limit problem (a hypothetical one from a physical point of view) reveals significant features of the behavior of the water table when $\mu_o$ T is small, and the method of proof will be indicated.

The limit value of the general term of the infinite series in Eq. 9(a) is

$$\mu_o T \rightarrow 0 \lim (2n + 1)^{-3} (-1)^n \left(1 - a_n\right) \exp - \mu_o T\,(2n + 1)^2 \gamma$$

$$= (2n + 1)^{-3} (-1)^n (1 - \theta) \ldots\ldots\ldots\ldots (32)$$

so that it follows from Eq. 9(a) that

$$h_p = \left(\frac{P_c L^2}{8}\right)\left\{1 - (1 - \theta)\left(\frac{32}{\pi^3}\right)\sum_{n=o}^{\infty} (2n + 1)^{-3} (-1)^n\right\} \ldots\ldots (33)$$

Because

$$\left(\frac{32}{\pi^3}\right)\sum_{n=o}^{\infty} (2n + 1)^{-3} (-1)^n = 1 \ldots\ldots\ldots\ldots (34)$$

it is found that

$$h_p = \left(\frac{P_c L^2}{8}\right)\theta \ldots\ldots\ldots\ldots (35)$$

The same result may be obtained from Eqs. 8(a) and 10(a). Using Eq. M.9 Eq. 5, for $\mu_o T \rightarrow 0$, yields the general expression

$$h_{p,x} = \left(\frac{P_c}{2}\right) x\ (L - x)\,\theta \ldots\ldots\ldots\ldots (36)$$

in which $\theta = t_1/T$ and $P_c = \frac{p}{K D} = \frac{(q/t_1)}{K D}$. It is noted that for intermittent recharge the rate of recharge $p = q/t_1$, in which $t_1 \leq T$. It is thus found, from Eq. 36 that

$$h_{p,x} = \left[\frac{\left(\frac{q}{T}\right)}{2 K D}\right] x\ (L - x), \text{ for } \mu_o T \rightarrow 0 \ldots\ldots\ldots (37)$$

which is the equation for the height of the water table above the water level in the drains for steady state flow (Eq. M.6), the continuous rate of recharge being $p_1 = q/T$.

The preceding derivation shows that, for small values of $\mu_o$ T, the water table will have an approximately parabolic shape when the ground water is

recharged intermittently and dynamic equilibrium has been reached. It also follows that the amplitude of fluctuation will be small compared to the average level of the water table, a feature brought out by numerical evaluation of Eq. 5 for low values of $\mu_o T$.

The transient Eq. 11 also leads to Eq. 37 for $\mu_o T \rightarrow 0$. From Eq. 2 is follows for the limit problem here under consideration that there would be no rise of the water table. This result indicates that it takes many periods T to approach equilibrium if $\mu_o T$ is small. The latter conclusion has important practical implications.

The previous considerations are not dependent on the value of $t_1$. In fact, the factor $t_1$ does not occur in the final result (Eq. 37). The preceding conclusions are therefore also valid for $t_1 \rightarrow 0$, that is, for intermittent instantaneous recharge.

*Double Periodic Recharge.*—In order to evaluate the effect of irrigation deep percolation losses on the water table, it is convenient to divide each year into two periods; the irrigation season and nonirrigation season. For the irrigation season, it will be assumed that the ground water is recharged at

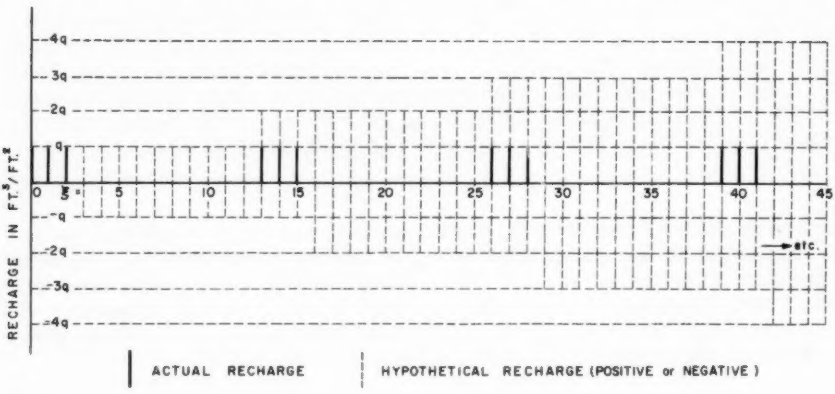



FIG. 2.—THREE RECHARGES q SPACED AT INTERVALS OF T = 28 DAYS REPEATED EVERY YEAR

regular intervals with every irrigation or with each alternate irrigation. During the nonirrigation season, there is presumably no recharge.

The problem of 'double periodicity' requires an extension of the theory of intermittent recharge, which is most easily explained by first analyzing the example shown in Fig. 2. It requires superposition of Eqs. 2 and M.38. The solution to the problem is obtained by superimposing hypothetical positive and negative recharges as explained in (MM),[2b] and illustrated in Fig. 2. (Attention is drawn to an error in the first paragraph of the second column on p. 557 of (MM). The second sentence reads "Let (r - 1) recharges be applied . . .," which should be corrected to read "Let r recharges be applied . . ."). In Fig. 2, three instantaneous recharges q are shown to be applied each year, spaced at 28-day ( = T) intervals, so that, for practical purposes, each year may be divided into thirteen 28-day intervals. Intermittent instantaneous recharge is depicted in the figure, but both intermittent instantaneous recharge and intermittent constant recharge will be considered in the following paragraphs.

The methodology to be developed requires that, for each particular problem, the irrigation season be divided into equal irrigation intervals and that the total year be expressed in a multiple of this interval. The latter is always possible because slight shortening or extension of the year does not significantly affect the result. The general problem will be treated later.

The problem shown in Fig. 2 is defined by Eq. 21 for values of $\xi \leq 3$ (in the first year). For $3 \leq \xi \leq 13$, the water table level is defined by Eq. M.38. In the second year, we need to evaluate the effect of the new recharges in that year as well as the residual effect of the recharges in the first year. In the third year, the effect of the recharges in that year needs to be evaluated in conjunction with those of the second and first years, and so forth for subsequent years. Water table rises will first be determined for the intervals $2 + 13(m - 1) \leq \xi \leq 3 + 13(m - 1)$, in which m = 1, 2, 3, . . . represents the year being considered.

The water table level in the first year, and for the preceding range of $\xi$, is given by Eq. 2

$$h_{x,1} = h_{px,\gamma} - h_{tx,2+\gamma} \quad \ldots\ldots\ldots\ldots\ldots (38)$$

in which the subscripts p, t, and x have their usual meanings; $\gamma$ represents the value to be used in the periodic function, and $(2 + \gamma)$ that to be used for $\xi$ in the transient. The subscript 1 of $h_{x,1}$ on the left hand side of Eq. 38 is to indicate that this water table level is that for the first year of recharge.

The effect on the water table in the second year of the three recharges applied during the first year is computed from Eq. M.38, that is,

$$h_{x,2} = h_{tx,12+\gamma} - h_{tx,15+\gamma} \quad \ldots\ldots\ldots\ldots\ldots (39)$$

in which the subscript 2 of $h_{x,2}$ indicates that this value represents the residual height of the water table in the second year, due to the recharges in the first year; the subscripts $(12 + \gamma)$ and $(15 + \gamma)$ on the right hand side represent values of $\xi$ to be used in Eq. 11 or Eq. M.30 for the transient.

In general, the effect on the water table level in later years of the three recharges applied during the first year may be evaluated from

$$h_{x,m} = h_{tx,\xi - 3} - h_{tx,\xi} \quad \ldots\ldots\ldots\ldots\ldots\ldots (40)$$

in which, for the $m^{th}$ year, $\xi = 13(m - 1) + 2 + \gamma$. Observe that Eq. 40 is valid for any year except the first year. This exception occurs because $h_{x,1}$ includes the periodic function in which the variable $\gamma$ can only vary over a unit interval of $\xi$, that is, from 0 to 1 (comment below Eq. 4). The height $h_{x,m}$ decreases monotonically, that is, its numerical value decreases as m increases and equals zero after an indefinitely large number of years. For the usual drainage situation, it is negligibly small after only a few years.

The resultant water table $h_{x,r}$ in any year, due to recharges in successive years, is obtained by simple addition of the water table rises as computed from Eqs. 38, 39, and 40. For the $m^{th}$ year it follows that

$$h_{x,r} = h_{x,1} + h_{x,2} + h_{x,3} + \ldots + h_{x,m-1} + h_{x,m} \quad \ldots\ldots (41)$$

The validity of this equation may be explained by redefining the symbols $h_{x,1}$, $h_{x,2}$, and so on. The symbol $h_{x,1}$ represents the water table rise due to recharges in the last $m^{th}$ year, $h_{x,2}$ is the water table rise due to recharges in the second last year, and, finally, $h_{x,m}$ is the water table rise in the $m^{th}$ year

due to recharges in the first year. The maximum height of the water table in the $m^{th}$ year is obtained from Eq. 41 by putting $\gamma = \xi$ for intermittent constant recharge, and by putting $\gamma = 0$ for intermittent instantaneous recharge.

Eq. 41 was derived for values of $\xi$ between $13(m - 1) + 2$ and $13(m - 1) + 3$ but may be extended to cover each whole irrigation season, that is, values of $\xi$ may vary from $13(m - 1)$ to $13(m - 1) + 3$. The variable $\gamma$ in the periodic function then goes from 0 to 1 as $\xi$ increases with each irrigation interval from one (positive) integral number to the next.

Similarly, we find for the resultant water table $h'_{x,r}$ during the nonirrigation season,

$$h'_{x,r} = h'_{x,1} + h'_{x,2} + h'_{x,3} + \ldots + h'_{x,m-1} + h'_{x,m} \quad \ldots\ldots\ (42)$$

in which, for $3 + 13(m - 1) \leq \xi \leq 13m$,

$$h'_{x,m} = h'_{tx,\xi-3} - h'_{tx,\xi} \quad \ldots\ldots\ (43)$$

The symbol $h'_{x,1}$ of Eq. 42 represents the water table rise due to recharges in the $m^{th}$ year, $h_{x,2}$ is the water table rise due to recharges in the $(m - 1)^{th}$ year, and so on. The subscript m of $h'_{x,m}$ indicates the last year considered in Eq. 42. It may also be taken to indicate the $\xi$ that is to be used in any one term (including the first term) of the series on the right hand side of Eq. 42. Any additional year requires one additional term in the series. The minimum height of the water table at the end of the first nonirrigation season is obtained from $h'_{x,1}$ by putting $\xi = 13$. The resultant water table $\left(h'_{x,r}\right)$ at the end of the $m^{th}$ nonirrigation season is computed from Eq. 43 by using $\xi = 13m$ for $h'_{x,m}$, $\xi = 13(m - 1)$ for $h_{x,(m-1)}$, . . . , $\xi = (13)\ (2)$ for $h'_{x,2}$, and, finally, $\xi = 13$ for $h'_{x,1}$.

The preceding values of $\xi$ may be generalized to any number of recharges during the irrigation season spaced at any interval. Let a recharges be applied and let the year be divided into b intervals of T days each. The irrigation season, or more appropriately, the season of recharge, then covers a intervals, and the nonirrigation season, the season without recharge, covers (b - a) intervals. Furthermore, let $\omega'$ be the 'time' within the year expressed in units of T, so that $\omega'$ varies from 0 (at the beginning of the irrigation season) to b (at the end of the nonirrigation season). It is convenient to have $\omega'$ further subdivided into $\omega + \gamma$, in which $\omega$ represents the integral numbers from 0 to b, while $\gamma$ varies from 0 to 1. The latter subdivision prevents confusion as to which value is to be used for $\gamma$ in the periodic function (Eqs. 5 or M.29) concurrently with the $\xi$'s for the other terms. For the irrigation season and for any $m^{th}$ year but the first year, the following result is obtained:

$$\begin{aligned} h_{x,r} = {} & h_{px,\gamma} - h_{tx,\omega+\gamma} \\ & + h_{tx,b-a+\omega+\gamma} - h_{tx,b+\omega+\gamma} \\ & + h_{tx,2b-a+\omega+\gamma} - h_{tx,2b+\omega+\gamma} \\ & + \ldots \\ & + h_{tx,(m-2)b-a+\omega+\gamma} - h_{tx,(m-2)b+\omega+\gamma} \\ & + h_{tx,(m-1)b-a+\omega+\gamma} - h_{tx,(m-1)b+\omega+\gamma} \quad \ldots\ (44) \end{aligned}$$

in which $0 \leq (\omega + \gamma) \leq a$. The maximum water table for any year is obtained by putting $\omega = (a - 1)$, and $\gamma = \zeta$ for intermittent constant recharge or $\gamma = 0$ for intermittent instantaneous recharge.

For the nonirrigation season it follows that

$$\begin{aligned} h'_{x,r} = {} & h_{tx,-a+\omega+\gamma} - h_{tx,\omega+\gamma} \\ & + h_{tx,b-a+\omega+\gamma} - h_{tx,b+\omega+\gamma} \\ & + h_{tx,2b-a+\omega+\gamma} - h_{tx,2b+\omega+\gamma} \\ & + \dots \\ & + h_{tx,(m-2)b-a+\omega+\gamma} - h_{tx,(m-2)b+\omega+\gamma} \\ & + h_{tx,(m-1)b-a+\omega+\gamma} - h_{tx,(m-1)b+\omega+\gamma} \quad \dots\dots (45) \end{aligned}$$

in which $a \leq (\omega + \gamma) \leq b$. The minimum water table for any year is obtained by putting $\omega + \gamma = b$.

Observe that all (except the first) terms of the right hand side of Eqs. 44 and 45 are identical in form. Furthermore, since the subscripts $(b - a + \omega + \gamma)$, and so on, represent values of $\xi$, it follows from Eqs. 11 and M.30 that the terms of both Eqs. 44 and 45 each form two geometric progressions (except the first term of Eq. 44 for the periodic function) which may be summed separately. Using Eqs. 11 or M.30 and performing the summations indicated, Eq. 44 yields, for the irrigation season, the following results:

1.—For intermittent constant recharge (Eqs. 4 and 9)

$$\begin{aligned} h_{x,e} = {} & h_{px,\gamma} - h_{tx,\omega+\gamma} + \left(\frac{P_c\ L^2}{8}\right)\left(\frac{32}{\pi^3}\right) \sum_{n=o}^{\infty} (2\ n + 1)^{-3} \sin (2n \\ & + 1)\, \pi \left(\frac{x}{L}\right) \left[\frac{\exp \mu_n\ Ta - 1}{\exp \mu_n\ Tb - 1}\right] a_n \exp - \mu_n\ T(\omega+\gamma) \quad \dots\dots\dots (46) \end{aligned}$$

of which $h_{px,\gamma}$ is computed directly from Eq. 5 and $h_{tx,\omega+\gamma}$ from Eq. 11; the maximum water table is obtained by putting $\omega = a - 1$, and $\gamma = \zeta$.

2.—For intermittent instantaneous recharge (Eqs. M.29 and M.30)

$$\begin{aligned} h_{ix,e} = {} & h_{pix,\gamma} - h_{tix,\omega+\gamma} + \left(\frac{q}{\epsilon}\right)\left(\frac{4}{\pi}\right) \sum_{n=o}^{\infty} (2\ n + 1)^{-1} \sin (2n \\ & + 1)\, \pi \left(\frac{x}{L}\right) \left[\frac{\exp \mu_n\ Ta - 1}{\exp \mu_n\ Tb - 1}\right] b_n \exp - \mu_n\ T(\omega+\gamma) \quad \dots\dots\dots (47) \end{aligned}$$

of which $h_{pix,\gamma}$ is computed directly from Eq. M.29 and $h_{tix,\omega+\gamma}$ from Eq. M.30; the maximum water table is obtained by putting $\omega = a - 1$, and $\gamma = 0$. The factor $b_n$ is given by Eq. 23. Eqs. 46 and 47 are valid for $0 \leq (\omega + \gamma) \leq a$. The notation in Eqs. M.29 and M.30 should be changed as indicated by Eq. 3.

Similarly, the following is found, for the nonirrigation season, from Eq. 45:

1. For intermittent constant recharge (Eqs. 5 and 11)

$$\begin{aligned} h'_{x,e} = {} & h_{tx,-a+\omega+\gamma} - h_{tx,\omega+\gamma} + \left(\frac{P_c\ L^2}{8}\right)\left(\frac{32}{\pi^3}\right) \sum_{n=o}^{\infty} (2\ n + 1)^{-3} \sin (2n \\ & + 1)\, \pi \left(\frac{x}{L}\right) \left[\frac{\exp \mu_n\ Ta - 1}{\exp \mu_n\ Tb - 1}\right] a_n \exp - \mu_n\ T(\omega + \gamma) \quad \dots\dots\dots\dots (48) \end{aligned}$$

of which the first two terms are computed directly from Eq. 11; the minimum water table is obtained by putting $(\omega + \gamma) = a$.

2. For intermittent instantaneous recharge (Eqs. M.29 and M.30)

$$h'_{ix,e} = h_{tix,-a+\omega+\gamma} - h_{tix,\omega+\gamma} + \left(\frac{q}{\epsilon}\right)\left(\frac{4}{\pi}\right)\sum_{n=o}^{\infty} (2\,n+1)^{-1} \sin (2\,n + 1)\,\pi \left(\frac{x}{L}\right)\left[\frac{\exp \mu_n\, Ta - 1}{\exp \mu_n\, Tb - 1}\right] b_n \exp - \mu_n\, T(\omega + \gamma) \qquad (49)$$

of which the first two terms are computed directly from Eq. M.30; the minimum water table is obtained by putting $(\omega + \gamma) = a$. Eqs. 48 and 49 are valid for $a \leq (\omega + \gamma) \leq b$.

Note that the notation on the left hand side of the preceding equations has been changed from $h_{x,r}$ and $h'_{x,r}$ to $h_{x,e}$ and $h'_{x,e}$, respectively. This is done because Eqs. 46, 47, 48, and 49 describe an equilibrium condition (theoretically for $m \rightarrow \infty$) equivalent to the so-called dynamic equilibrium defined by the periodic functions in (MM) for 'single periodic' recharge. The transients have been omitted because the ultimate equilibrium water table level is of primary interest in open ditch or tile drainage problems. The water table levels midway between the drains $(h_e$ and $h'_e)$ are determined from Eqs. 46, 47, 48 and 49 by putting $(-1)^n$ for $\sin (2\,n + 1)\,\pi\,(x/L)$. For $\omega + \gamma = a$, Eqs. 46 and 48 are identical and so are Eqs. 47 and 49. Eq. 46 with $\omega + \gamma = 0$ is identical to Eq. 48 with $\omega + \gamma = b$. Eq. 47 with $\omega + \gamma = 0$ is identical to Eq. 49 with $\omega + \gamma = b$ if the water table rise due to one recharge $(q/\epsilon)$ is added to the latter equation.[2c]

Equations for the rate of discharge are not given but may be obtained in exactly the same manner. In fact, the equations for 'double periodic' intermittent constant recharge may be immediately written down from Eqs. 14(a) and 18 from inspection of the foregoing equations.

## CONCLUSIONS

Of all the equations presented in the paper, those for intermittent instantaneous recharge are the most useful for practical work. Their greater simplicity makes them attractive. Furthermore, numerical evaluation shows that there is often little difference between water table levels computed for an interval of recharge that is either finite or zero, providing the amount of recharge is the same. Another important reason for their usefulness is that, in irrigated agriculture, recharge to the groundwater approaches the condition of instantaneous recharge in many cases. The word 'approach' is used advisedly; instantaneous recharge, as defined mathematically, obviously can never take place in nature. Field observations, for which data have been gathered by Dumm, have shown that this recharge condition is often closely approximated in normal irrigation practice. Comprehensive theoretical studies on flow of water through unsaturated soil support the field observations.

There are, of course, problems for which the interval of recharge must be kept finite in order to obtain reasonably accurate results. Water table fluctuations in a slowly permeable overburden (and consequently with low drainable porosity) require careful consideration, as do the discharges of drainage systems. Furthermore, if one wishes to compute the height of the

water table in a slowly permeable overburden (Fig. 3) rather than the piezometric head in the aquifer,[2a] it becomes necessary to take into account its hydraulic conductivity, $K_1$. The effect on the water table of an overburden with a low hydraulic conductivity in the vertical direction has been ignored in the present study. The solution of that problem requires an extension of the theory to include evaluation of hydraulic head losses in the slowly permeable stratum. It then becomes necessary to solve two simultaneous differen-

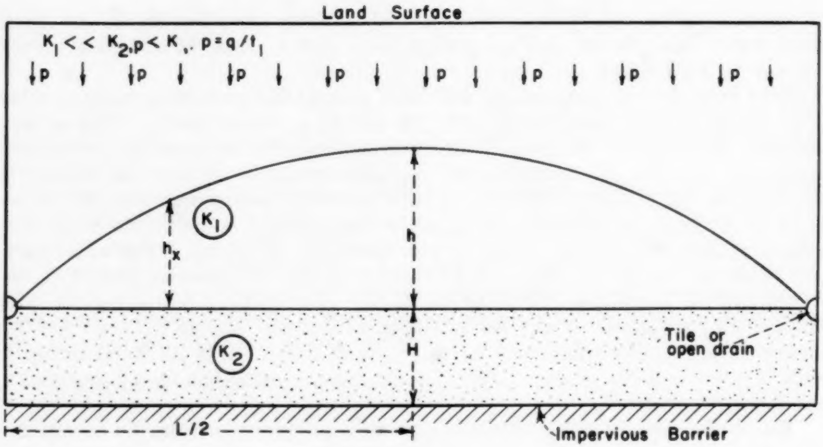



FIG. 3.—FLOW THROUGH A HORIZONTAL HIGHLY PERMEABLE LAYER

tial equations as is done by J. Wesseling to determine[22] the effect on the water table of tidal fluctuations in a river.

## ACKNOWLEDGMENTS

Most of the work reported in this paper was done by the writer while he was employed by the U. S. Bureau of Reclamation, Dept. of Interior (USBR). Earlier technical memorandums on this subject prepared by Glover were of great value to the writer in his study of the problem. The encouragement of Messrs. P. H. Berg, M. ASCE, and J. T. Maletic is gratefully acknowledged. Conversations with Messrs. Glover, Dumm, and Tapp were stimulating and helpful.

---

22 "The Tranmission of Tidal Waves in Elastic Artesian Basins," by J. Wesseling, Netherlands Journal of Agricultural Science, Vol. 7, 1959, pp. 22-32.

## APPENDIX.—NOTATION

The symbols adopted for use in this paper are listed here for ease of reference and for the use of discussers:

$a$ = total number of recharges during one recharge season in problems involving 'double periodic' recharge; the length of the recharge season is then $aT$;

$b$ = number of intervals $T$ into which each year is divided in problems involving 'double periodic' recharge;

$D$ = weighted mean depth of the profile through which the flow occurs;

$D_T$ = $1/2qL$, total volume of groundwater released per period $T$ from one side of the drain, after dynamic equilibrium has been reached;

$h$ (or $h_p$, $h_{p1}$, $h_{ph}$, $h_{pi}$, $h_{pi,1}$, $h_{pi,h}$, $h_t$, $h_{ti}$) = height of the water table midway between drains, measured from the line connecting the water levels in the drains; the subscripts 1 and h are used to denote the minimum and maximum water table levels, respectively;

$h_{p,x}$ (or $h_{pi,x}$) = height of the water table for any $x$, as computed from the periodic functions for 'single periodic' recharge;

$h_{t,x}$ (or $h_{ti,x}$) = height of the water table for any $x$, as computed from the transient functions for 'single periodic' recharge;

$h_x$ (or $h_{i,x}$) = height of the water table for any $x$, measured from the line connecting the water table levels in the drains;

$h_{x,r}$ (or $h'_{x,r}$) = resultant height of the water table after a number of years of 'double periodic' recharge; the symbol without prime denotes the water table during the recharge season, with prime it denotes the water table during the non-recharge season;

$h_{x,m}$ (or $h'_{x,m}$) = the rise of the water table in the $m^{th}$ year due to recharges in first year, as computed for 'double periodic' recharge; the symbol without prime denotes the water table rise during the recharge season, with prime it denotes the rise during the non-recharge season;

$h_{x,e}$ (or $h'_{x,e}$, $h_{ix,e}$, $h'_{ix,e}$) = height of the water table as computed for the condition of dynamic equilibrium from the periodic functions for 'double periodic' recharge;

$H$ = depth of permeable stratum below the line connecting the water levels in the drains;

$K$ = lateral hydraulic conductivity;

$L$ = width of aquifer or spacing of drains;

$p$ = $q/t_1$, constant rate of surface-applied recharge per unit area of land surface;

$P_c$ = $p/KD$, recharge coefficient for $L \gg D > h_m$ and $p \ll K$;

$q$ = amount of surface-applied recharge per unit area of land surface;

$Q$ (or $Q_p$, $Q_{p,min}$, $Q_{p,max}$, $Q_t$, $Q_i$, $Q_{pi}$, $Q_{ti}$) = rate of discharge per unit length of drain computed for flow into the drain from only one side of the groundwater reservoir;

$r$ = number of recharges considered;

$S_i$ (or $S_{pi}$, $S_{ti}$) = volume of groundwater stored below the water table per unit width of aquifer and between two drains spaced at a distance L; groundwater storage has been considered only for intermittent instantaneous recharge;

$t$ = time since the beginning of the first recharge;

$t_1$ = interval of recharge within the period T;

$T$ = time interval between the beginning of consecutive recharges (in days);

$x$ = horizontal coordinate;

$\epsilon$ = drainable porosity or specific yield;

$\mu_n$ = $(K D/\epsilon)\ (2n + 1)^2\ (\pi/L)^2$;

$\mu_o T$ = $(K D/\epsilon)\ (\pi/L)^2\ T$, the aquifer response coefficient;

$\pi$ = 3.14159;

$\tau$ = time measured within the period T; $0 \leq \tau \leq T$; and

$\omega'$ = $\omega + \gamma$, 'time' within the year expressed in units of T; $\omega'$ varies from 0 (at beginning of the recharge season) to b (at end of non-recharge season).

The symbols $\gamma$, $\zeta$, $\delta$, $\xi$ are defined by Eq. 2, $\alpha(\gamma)$ and $\beta(\gamma)$ by Eq. 3, $a_n$ by Eq. 5, and $b_n$ by Eq. 21; they are all dimensionless. The subscript i is used when instantaneous recharge is assumed.

Journal of the

IRRIGATION AND DRAINAGE DIVISION

Proceedings of the American Society of Civil Engineers

# DISCUSSION

Note.—This paper is a part of the copyrighted Journal of the Irrigation and Drainage Division, Proceedings of the American Society of Civil Engineers, Vol. 87, No. IR 2, June, 1961.

# WORLD PRACTICES IN WATER MEASUREMENT AT TURNOUTS[a]

Closure by Charles W. Thomas

CHARLES W. THOMAS,[18] F. ASCE.—The discussion by Chow serves to illustrate and emphasize the magnitude of the problems associated with control and measurement of water at the farm turnout.

TABLE 1.—EVALUATION TABLE FOR WATER MEASUREMENT DEVICES

| | Description | | Hydraulic Properties | | | |
|---|---|---|---|---|---|---|
| Device or method | Operating principle and function | Relative size | Type | Head loss | Range of operation | Expected accuracy |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) |

| Cost | | | | Operational Problems | | | |
|---|---|---|---|---|---|---|---|
| | | | | Fouling by | | | |
| Construction | Operation | Maintenance | Estimated life | Weeds etc. | Sediment | Auxiliary equipment necessary | Remarks |
| (8) | (9) | (10) | (11) | (12) | (13) | (14) | (15) |

It is regrettable that time and space die not permit the writer to evaluate each of the devices mentioned in the paper. A very useful contribution to the profession would be a summary of all known measurement devices and methods. Such a summary, preferably in tabular form, giving for each method and device, such information as: basic operating principles, whether used in conjunction with, or independent of a control device, range of flow rates that can be measured, head loss, cost, and so forth, should be very useful if used with care. The headings for such a table might be similar to that given in Table 1. Perhaps this suggestion could be the stimulus for someone to expend time and effort to produce such an evaluation chart or charts.

It is particularly gratifying to the writer that Chow mentioned the desirability of having a single structure that will serve the dual function of control, regulation, and measurement of flows. As mentioned in the paper a shutoff is, in most instances, required at a farm delivery because of the manner in which

---

[a] June 1960, by Charles W. Thomas (Proc. Paper 2530).

[18] Hydr. Engr., Bur. of Reclamation, U.S. Dept. of the Interior, Denver, Colo.

the systems are operated. Economy in installation can be effected if the shut-off also serves as a regulator and as a means of measurement. Many times it may be economical or convenient to utilize control devices and structures as indications of the quantity of water flowing. Such structures are for the most part designed without thought of their use for this purpose. The difficulties of adapting them may be many, but the information resulting from calibration may be adequate for operational purposes and, in some instances, may be of a high order of accuracy. It must first be determined how well the control will serve for measurement and the expected accuracy of results. A calibration may be obtained by application of data derived from similar devices, by hydraulic model tests, or by measurements made in the field. Regardless of the method used to obtain a calibration of a control device, the accuracy of the calibration can be no better than the accuracy of the device or means used to effect the calibration. It may be seen then that caution is indicated in making such calibrations, and the results should be critically judged before general usage.

Many operational objections can be removed if a single device can be made to serve the two functions, control and measurement of flows.

# LOS ANGELES WATER SUPPLY AND IRRIGATION[a]

Discussion by William W. Donnan and Charles H. Lee

WILLIAM W. DONNAN,[2] F. ASCE.—Of particular interest to this writer was to follow the progress and note that only by group action was it possible to plan and finance the engineering works required to bring water to the Los Angeles, California, area.

California laws and policies create a favorable climate for the organization of water districts to cope with and resolve water problems. Today (1961) there are over 2,500 municipal, commercial, mutual or other organized water service agencies in California and over 1,000 are located in Southern California.[3]

If Southern California is to continue to expand in population and industrial growth, additional water must be developed and imported. The State Department of Water Resources estimates that, at the present time (1961), there is a need for about 400,000 acre ft of additional water per yr.[3] In looking to the future, if all the agricultural, urban and industrial expansion were to continue to an ultimate level, the mean seasonal requirement of additional water would be 4,000,000 acre ft.[3] How can this challenge be met?

The solution to the water problems of the future rest in the capacity of Southern California's people to cooperate. There are a number of different schemes being investigated to develop new sources of water. Each one has certain merits but each one will require the utmost in cooperation, grouping of effort and teamwork. They are as follows:

1. The conversion of sea water involves the purification of sea water so that it can be used for domestic and irrigation purposes. The best known methods utilize tremendous amounts of electric current. Moreover, any water development will have to be pumped from sea level to its point of use. At present only large-scale, high initial investment plants offer the promise of an economical enterprize. All these provisions require a high degree of give and take between industry, community and agriculture.

2. The Feather River Project[4] involves building a huge dam at Oroville on the Feather River. The proposed capacity of this reservoir would be 3,500,000 acre ft and the dam would be 710 ft high. There would be installed electric generators to provide power for pumping the water. The water would flow down the Sacramento River and then be pumped up to canals and conduits flowing southward toward Los Angeles. The entire scheme would cost about 1.5 billion

---

a December 1960, by Samuel B. Morris (Proc. Paper 2671).

2 Agric. Engr., Soil and Water Conservation Research Div., Agric. Research Service, U. S. Dept. of Agric., Pomona, Calif.

3 "Water Utilization and Requirements of California," Bulletin No. 2, Publication of the Calif. State Water Resources Bd., June, 1955.

4 "The Feather River Plan," by A. D. Edmonston. Presented in March, 1956 to the ASCE at San Francisco, Calif.

dollars. It would require that Southern California cooperate with Central Valley and San Joaquin Valley people to vote bonds for this great enterprize. The initial stages of the Feather River Project are started.

3. The California Water Plan[5] involves the utilization of the total soil and water resources of the State. The plan calls for building 260 new major reservoirs and an aqueduct system to transport the water from the northern areas of excess to the arid south. It would cost an estimated 12 billion to 13 billion dollars. The Feather River Project would be only one facet of the total plan. This plan would require the utmost in cooperation between areas within the State. For example, northern water rights would have to be safeguarded, more metropolitan water might have to be sent to San Diego County and replaced with Feather River water, and water rights would have to be traded or pooled in Southern California.

On November 8, 1960 the people of California voted bonds for the development of the California Water Plan. Fortunately, California has an outstanding State Department of Water Resources. This technical staff can be counted on to provide all the skill and engineering science to develop these schemes.

The big question is, can the people, all the people, organize and cooperate to bring these schemes to fulfillment? History has shown that only when two or more farmers join together can they divert a stream; only when a group organizes can they build a reservoir; only when a large city cooperates can they build an aqueduct. Only when cities and groups of cities combine can they build a Colorado River Aqueduct.

Solution of these problems might be facilitated by the creation of a water authority for all of Southern California where water rights are pooled and resources are taxed for the development of water. This agency, working through the State Department of Water Resources, could negotiate with and join with the Northern California groups to ultimately develop the total water supplies of this state.

CHARLES H. LEE,[6] F. ASCE.—The author has presented, in a most interesting manner, the early history of the water supply of the City of Los Angeles. This history is of particular interest to the writer as he was, for many years (1906 to 1935), intimately connected with the activities of the Los Angeles Water Department and especially with the Owens River Aqueduct.

Los Angeles, being in a semi-arid region, has had to seek water from great distances to supply the needs of its rapidly growing population. As stated by the author, the early policy of the Water Department was not to develop nearby sources of water needed for the growth of neighboring communities. This led to the selection of the Owens River, at a distance of 250 miles, and later the Mono Basin, at a distance of 338 miles, and the Colorado River, 350 miles away, as sources of supply. Continuing increase in demand is presently causing consideration of the use of water from the Feather River in Northern California, a distance of 420 miles. It is noteworthy, however, that this policy has not been extended to the "strangers within the gates" of the San Fernando Valley.

The Los Angeles Department of Water and Power, during its history, has pioneered many new ideas in engineering research and procedure, as well as

[5] "Preview of the California Water Plan," publication of the State Water Resources Bd., March, 1956.

[6] Senior Partner, Lee and Praszker, Cons. Civ. Engrs., San Francisco, Calif.

of construction methods. During the Owens River Aqueduct period, the use of tufa cement was developed and practically applied. Much was also contributed to the mechanics of powered transportation over rough terrain. Early in 1908, Chief Engineer William Mulholland initiated the first quantitative survey of inflow and outflow from a closed ground water basin. This was in the Owens Valley where he recognized the great potentiality of the Independence Basin as a reserve source of supply for the Aqueduct in dry years. The writer was privileged to conduct this survey,[7] that extended over a period of several years, and led to extensive well development, both in the Independence region and the upper portion of Owens Valley in the Big Pine and Bishop regions. During the dry period between 1920 and 1930, this source was of material aid in maintaining the Aqueduct supply. The basic hydrologic methods followed in making this survey have today been developed into standard, more or less routine, methods of arriving at hydrologic inventories and safe yield values for closed ground water basins. Such methods are in use not only among private engineers but also the engineer-geologists of the Ground Water Division of the U. S. Geological Survey and of various state agencies, such as the Water Resources Department and the Water Rights Board of the State of California.

Another field of hydrologic engineering pioneered by the Department was the making of snow surveys in mountain watersheds for the purpose of estimating available seasonal water supply. This work was initiated about 1911 by J. E. Jones, M. ASCE, Chief Hydrographer of the Department of Water and Power, and technique was developed in cooperation with other agencies working on the east slope of the Sierra Nevada Mountains. Such surveys later became routine practice throughout the Sierra Nevada and other high mountain ranges of the West. In California and other western states, snow surveys are now conducted annually by State agencies, supplemented in some areas by the Federal government.

During the succeeding years, the Department has initiated and pioneered many other technical engineering procedures, standardized designs, and construction aids that have placed it among the leaders in the waterworks field.

The paper ends with the statement that the irrigation chapter within the City of Los Angeles is about to close. The impact of this statement on the ground water supply of the San Fernando Valley is notable, because urbanization with paved areas and roofs is taking the place of irrigation. Furthermore, demand has arisen for flood control on streams traversing the valley. These improvements greatly reduce the opportunity for natural percolation to the water table by absorption of direct rainfall on the valley floor and especially of seepage from runoff in stream channels that have now been largely sealed by impervious concrete lining. The effect of these improvements in depletion of the natural safe yield of the basin is substantial.

The author is to be congratulated for his interesting recording of the historical highlights of one of the outstanding waterworks systems of the United States.

---

7 "The Determination of Safe Yield of Underground Reservoirs of the Closed Basin Type," by Charles H. Lee, Transactions, ASCE, Vol. LXXVIII, 1915.

# METHOD FOR ESTIMATING CONSUMPTIVE USE OF WATER FOR AGRICULTURE[a]

Discussion by George H. Hargreaves

GEORGE H. HARGREAVES,[15] F. ASCE.—Many methods are in use for computing evapotranspiration from climatic data. Most of these, although useful in certain geographic areas, are either approximations or limited in suitability for general application. Three methods appear to offer the advantages of accuracy and general applicability. These are the Penman Method, the black bulb evaporation atmometer and the evaporation pan. Of these three the evaporation pan offers the advantages of more general availability of data and direct relationship to evapotranspiration. A summary of some of the literature describing this relationship is given by Pruitt[12] in his introduction.

Available research data indicate that irrigation requirements or consumptive use requirements depend primarily on the degree of ground cover and on the rate and stage of growth. Rapidly growing crops are usually dark green and consequently are able to absorb a maximum of solar energy. A dark green crop providing complete ground cover will have a maximum potential consumptive use or potential evapotranspiration. This concept of potential evapotranspiration is used by Penman, Thornthwaite, and Pruitt.

In order to relate climatic factors to consumptive use or irrigation requirements, it would seem desirable that the correlation between the factor and potential evapotranspiration be evaluated. Pruitt measured potential evapotranspiration by clover at Prosser, Washington and correlated the results with evaporation from six types of pans. Coefficients of correlation for the evaporation pans varied from 0.972 to 0.990. The coefficient of correlation with black bulb atmometer evaporation was found to be 0.972 and with Penman's formula ($Eo \times 0.97$) to be 0.986. For the Blaney-Criddle method ($K = 1.08$) the coefficient was found to be 0.908, and for Thornthwaite's method ($E\ T \times 1.72$) it was found to be 0.849. The data and discussion presented by Pruitt clearly support the use of pan evaporation data in determining consumptive use or irrigation requirements.

If it is concluded that pan evaporation data give an accurate index of the consumptive use of water by agricultural crops, then a comparison of pan evaporation with results from any given method of computation provide a means of evaluating that particular method.

Munson's method of estimating consumptive use from temperature data by use of Table 1 can be evaluated by using a climatic atlas in order to obtain an overall visual representation of climatic conditions. Fig. 2 shows temperature and evaporation conditions for the United States during January. Fig. 3 shows

[a] December 1960, by Wendell C. Munson (Proc. Paper 2672).

[15] Irrig. Advisor ICA, Rio de Janeiro, Brazil.

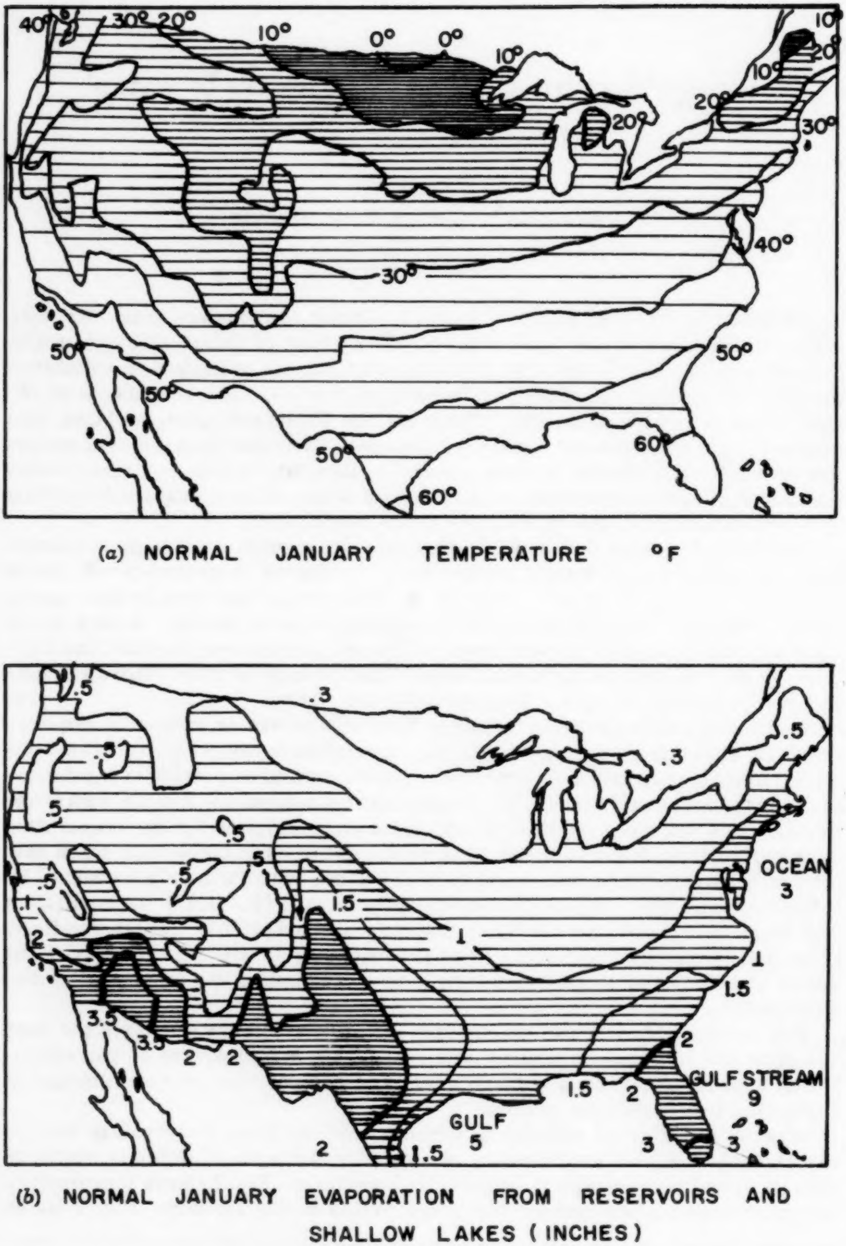
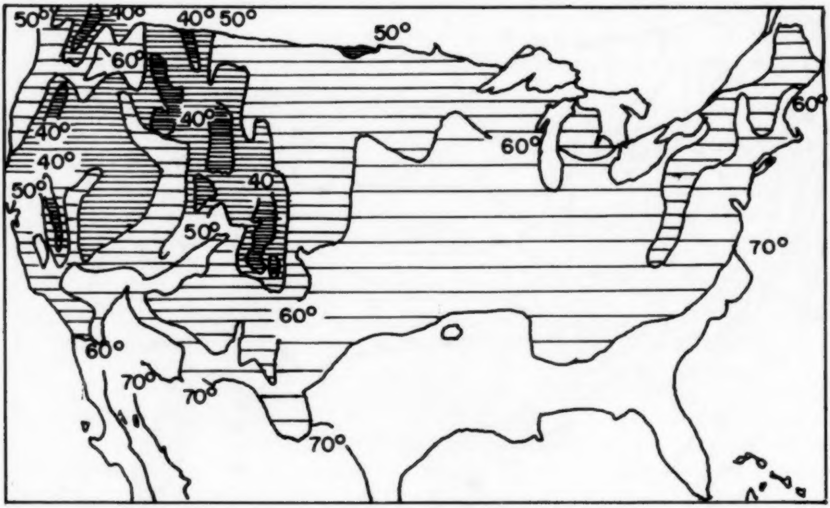
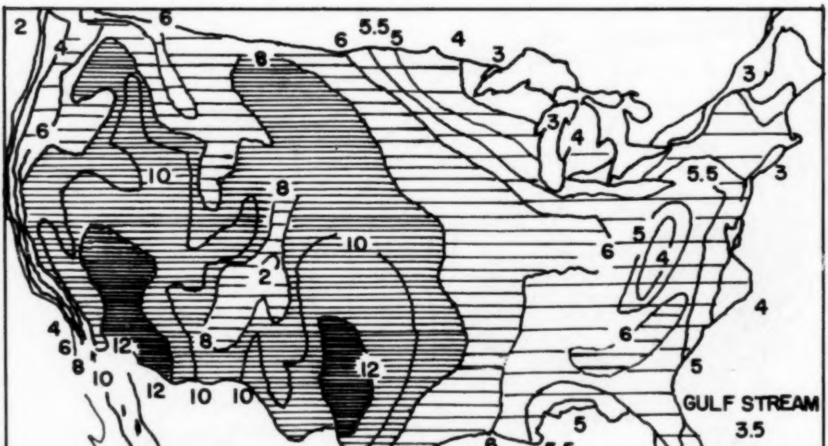



FIG. 2.—COMPARISON OF JANUARY TEMPERATURE AND EVAPORATION

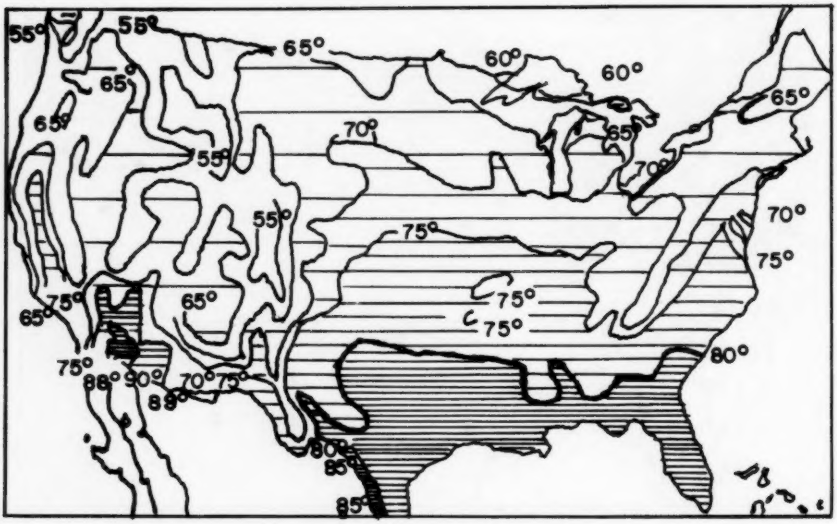



(*a*) NORMAL JULY DAILY MINIMUM TEMPERATURE °F

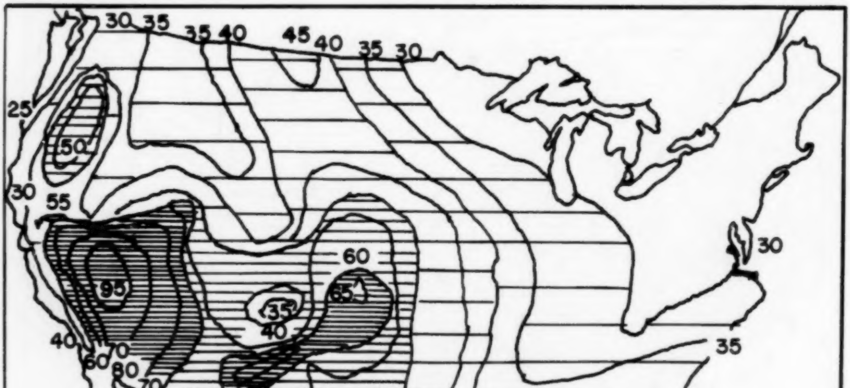



(*b*) NORMAL JULY EVAPORATION FROM RESERVOIRS AND SHALLOW LAKES (INCHES)

FIG. 3.—COMPARISON OF JULY TEMPERATURE AND EVAPORATION




(a) NORMAL SUMMER TEMPERATURE °F




(b) NORMAL MAY – OCTOBER EVAPORATION FROM PANS - (INCHES)

FIG. 4.–COMPARISON OF SUMMER TEMPERATURE WITH GROWING SEASON EVAPORATION

conditions during July. The climatic data used in the figures are from Stephen S. Visher.[16] Evaporation data used in Figs. 2 and 3 are from reservoirs and shallow lakes and are approximately proportional to pan evaporation. Fig. 4 shows normal May through October evaporation from pans and normal summer temperatures.

Blaney[10] shows monthly ratios of consumptive use of water by alfalfa to pan evaporation. These vary from 0.45 to 0.86 with an average value of 0.66. Low rates of use by alfalfa occurred after cutting of the crop thus reducing the total water requirement. Pruitt[12] measured the consumptive use of water by Ladino clover on two plots cut at staggered times so that one plot would always provide 100% ground clover and consequently full potential evapotranspiration. Consumptive use or evapotranspiration under these conditions was found to be 108% of evaporation from a U. S. Weather Bureau Class A pan. By selecting a suitable factor to represent the degree of ground cover, evaporation, as shown in Fig. 4, provides a convincent index of the consumptive use of water for agriculture.

Figs. 2, 3, and 4 indicate that variations in evaporation and consequently of consumptive use are considerably more complex than variations in temperature. An inspection of Fig. 3, indicates a fairly large area in the southern portion of the United States with approximately the same mean monthly temperature. Evaporation within this area of approximately the same temperature varied from a low of 5 in. in the more humid areas to a high of 12 in. or more in the arid areas. Although temperature may provide a convenient and useful index of the consumptive use of water for some areas, the method proposed by Munson should be used with considerable caution in others.

*Errata.*—Two typographical errors should be corrected. On page 54, line 6, black atmometers absorb more than 90% of the solar energy instead of 50%. At the bottom of Table 7 (page 55) the reference should be 12 instead of 10.

---

16 "Climatic Atlas of the United States," by Stephen S. Visher, Harvard Univ. Press, Cambridge, Mass., 1954.

# IRRIGATION SYSTEMS OF THE TIGRIS AND EUPHRATES VALLEYS[a]

Discussion by Vahe J. Sevian and Kamil Taj-Eddin

VAHE J. SEVIAN,[3] F. ASCE and KAMIL TAJ-EDDIN.[4]—Varieties of wheat and barley were cultivated in the northern regions of Mesopotamia 60 centuries B.C. Agricultural life spread in central and southern Mesopotamia 20 centuries later, whereas after a further period of 20 centuries (bringing us to circa 2000 B.C.) greater varieties of land products were obtained. Rice followed much later at about 3 centuries B.C.

Without entering into the controversial question on the "Garden of Eden," or the "Hanging Gardens of Babylonia," the prosperity of the valleys continued and attained its height under the brillant achievements of the Abbasids (754 through 1258 A.D.), ending with the great blow given by Hulagu in 1258 A.D. The next few centuries, intermittent wars and apathy by the rulers were not helpful in restoring lost prosperity.

At the turn of the present century, efforts were noticeable for improving irrigation agriculture. Progress, though slow, was appreciable. From mid-century onwards greater stress is made in improving the agricultural pattern through modern technics, by making better use of the water, by restoring to the soil in many regions its lost productivity, and without neglecting the social aspects of the various problems involved.

The author mentions that "successful development of irrigation in the valley . . . is greatly dependent on good engineering, progressive laws . . . and cooperation with neighbouring countries." These are obvious; an important point is the need of an integrated program of development of the main resources: land and water involving human association.

Development of water resources will imply understandings with countries on which the valley depend for their supplies. These were not needed during past centuries, for the basin was almost entirely within one country, the Ottoman Empire and moreover, water requirements were adequate to conditions prevailing then. With the end of First World War, Syria and Iraq ceased to be parts of the Ottoman Empire. Consequently, the necessity for understandings is now felt with serious considerations given for agricultural and other developments by all the riparian states.

Since July 1958, the country is amidst an "Agrarian Reform" process. This will change the thousands of years practices and land (agricultural) tenure into a new aspect more adaptable to advanced conditions.

Throughout history, the "Great Flood" was followed by very many others. Their destructive effects were appraised by the extent of damages done to rural

[a] December 1960, by Stanley S. Butler (Proc. Paper 2673).

[3] Formerly Inspector General of Irrig., Ministry of Agric., Baghdad, Iraq.

[4] Engr.-Secry. to the Dir. Gen. of Irrig., Baghdad, Iraq.

and urban settlements in their varying degree of advancement. In recent years the Euphrates, in 1940, and the Tigris, in 1946 and 1954, caused very serious flood damages estimated at $200,000,000 for the 1954 flood mentioned in the paper.

Since 1941, floods on the Euphrates valley are sufficiently controlled by works at Habbaniya. On the Tigris flood risks are almost eliminated through the Tharthar project completed just after the 1954 flood.

Tidal irrigation has for centuries been practiced in the estuary of the rivers. Further upstream, gravity flood and flood irrigation was applied. This process is still practiced but greatly improved by means of water diversional structures and methods. There is another method of irrigation, with over 5,000 small individual pumps erected along the rivers and some canals. In the northern semi-humid regions, irrigation depends on the vagaries of rainfall.

A method timidly used till now is the utilization on a greater scale of the ground water resources wherever suitable in quantity and quality in regions distant from main streams. Investigations and surveys were undertaken and the author's contribution in this respect is acknowledged.

As regards to ultimate possibilities of irrigation with reference to surface and ground waters, it seems advisable to ascertain first volumes to be withdrawn from the rivers outside Iraq and then to decide on the future agricultural pattern and probable change either totally or regionally from the present fallow to intensive cultivation.

Dam sites exist in Iraq and in the headwaters of the rivers. Though the depression of Abu Dibbes on the Euphrates and Tharthar on the Tigris have been mentioned for flood control and storage, it will after proper investigations appear that other possibilities will afford better results for water conservation purposes.

On the Euphrates, Turkey's demands will mostly be for hydro-electricity. The retention of the water in the reservoirs and subsequent releases may not suit the agricultural requirements of distant Iraq, unless such releases are harmonized through a reservoir preferably in Iraq. In Syria irrigation requirements have priority over hydro-electricity to a certain extent.

The author mentions - from personal knowledge - that "in several areas bordering Iran there is often a severe shortage of surface water for irrigation in Iraq." The water is used for areas across the border with disregard to the interests and vested rights from time immemorial of the frontier regions of Iraq. Understanding between the interested countries is essential, ground water will not satisfy the requirements of the affected areas, only water diversional works from the Diyala river will help in satisfying the region. Recently the flow of a stream passing through Khanaqin (a frontier rural and urban centre) was entirely cut before crossing the frontier.

Water rights to individuals agricultural plots are defined and usually distributed in canals controlled by the Irrigation Department in strict proportion to cultivable areas. In some ancient canals still in operation, old established rights are maintained for the time being. Years are needed to change a system used during centuries and adopt new methods.

The writers confidently augur that during a short period of years progress in many fields has been achieved in a small region of the world when irrigation has been practiced for over 60 centuries.

## PROCEEDINGS PAPERS

The technical papers published in the past year are identified by number below. Technical-division sponsorship is indicated by an abbreviation at the end of each Paper Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Pipeline (PL), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways and Harbors (WW), divisions. Papers sponsored by the Department of Conditions of Practice are identified by the symbols (PP). For titles and order coupons, refer to the appropriate issue of "Civil Engineering." Beginning with Volume 82 (January 1956) papers were published in Journals of the various Technical Divisions. To locate papers in the Journals, the symbols after the paper number are followed by a numeral designating the issue of a particular Journal in which the paper appeared. For example, Paper 2703 is identified as 2703(ST1) which indicates that the paper is contained in the first issue of the Journal of the Structural Division during 1961.

### VOLUME 86 (1960)

JUNE: 2494(IR2), 2495(IR2), 2496(ST6), 2497(EM3), 2498(EM3), 2499(EM3), 2500(EM3), 2501(SM3), 2502(EM3), 2503(PO3), 2504(WW2), 2505(EM3), 2506(HY6), 2507(WW2), 2508(PO3), 2509(ST6), 2510(EM3), 2511(EM3), 2512(ST6), 2513(HW2), 2514(HY6), 2515(PO3), 2516(EM3), 2517(WW2), 2518(WW2), 2519(EM3), 2520(PO3), 2521(HY6), 2522(SM3), 2523(ST6), 2524(HY6), 2525(HY6), 2526(HY6), 2527(IR2), 2528(ST6), 2529(HW2), 2530(IR2), 2531(HY6), 2532(EM3)[c], 2533(HW2)[c], 2534(WW2), 2535(HY6)[c], 2536(IR2)[c], 2537(PO3)[c], 2538(SM3)[c], 2539(ST6)[c], 2540(WW2)[c].

JULY: 2541(ST7), 2542(ST7), 2543(SA4), 2544(ST7), 2545(ST7), 2546(HY7), 2547(ST7), 2548(SU2), 2549(SA4), 2550(SU2), 2551(HY7), 2552(ST7), 2553(SU2), 2554(SA4), 2555(ST7), 2556(SA4), 2557(SA4), 2558(SA4), 2559(ST7), 2560(SU2)[c], 2561(SA4)[c], 2562(HY7)[c], 2563(ST7)[c].

AUGUST: 2564(SM4), 2565(EM4), 2566(ST8), 2567(EM4), 2568(PO4), 2569(PO4), 2570(HY8), 2571(EM4), 2572(EM4), 2573(EM4), 2574(SM4), 2575(EM4), 2576(EM4), 2577(HY8), 2578(EM4), 2579(PO4), 2580(EM4), 2581(ST8), 2582(ST8), 2583(EM4)[c], 2584(PO4)[c], 2585(ST8)[c], 2586(SM4)[c], 2587(HY8)[c].

SEPTEMBER: 2588(IR3), 2589(IR3), 2590(WW3), 2591(IR3), 2592(HW3), 2593(IR3), 2594(IR3), 2595(IR3), 2596(HW3), 2597(WW3), 2598(IR3), 2599(WW3), 2600(WW3), 2601(WW3), 2602(WW3), 2603(WW3), 2604(HW3), 2605(SA5), 2606(WW3), 2607(SA5), 2608(ST9), 2609(SA5)[c], 2610(IR3), 2611(WW3)[c], 2612(ST9)[c], 2613(IR3)[c], 2614(HW3)[c].

OCTOBER: 2615(EM5), 2616(EM5), 2617(ST10), 2618(SM5), 2619(EM5), 2620(EM5), 2621(ST10), 2622(EM5), 2623(SM5), 2624(EM5), 2625(SM5), 2626(SM5), 2627(EM5), 2628(EM5), 2629(ST10), 2630(ST10), 2631(PO5)[c], 2632(EM5)[c], 2633(ST10), 2634(ST10), 2635(ST10)[c], 2636(SM5)[c].

NOVEMBER: 2637(ST11), 2638(ST11), 2639(CO3), 2640(ST11), 2641(SA6), 2642(WW4), 2643(ST11), 2644(HY9), 2645(ST11), 2646(HY9), 2647(WW4), 2648(WW4), 2649(WW4), 2650(ST11), 2651(CO3), 2652(HY9), 2653(HY9), 2654(ST11), 2655(HY9), 2656(HY9), 2657(SA6), 2658(WW4), 2659(WW4)[c], 2660(SA6), 2661(CO3), 2662(CO3), 2663(SA6), 2664(CO3)[c], 2665(HY9)[c], 2666(SA6)[c], 2667(ST11)[c].

DECEMBER: 2668(ST12), 2669(IR4), 2670(SM6), 2671(IR4), 2672(IR4), 2673(IR4), 2674(ST12), 2675(EM6), 2676(IR4), 2677(HW4), 2678(ST12), 2679(EM6), 2680(ST12), 2681(SM6), 2682(IR4), 2683(SM6), 2684(SM6), 2685(IR4), 2686(EM6), 2687(EM6), 2688(EM6), 2689(EM6), 2690(EM6), 2691(EM6)[c], 2692(ST12), 2693(ST12), 2694(HW4)[c], 2695(IR4)[c], 2696(SM6)[c], 2697(ST12)[c].

### VOLUME 87 (1961)

JANUARY: 2698(PP1), 2699(PP1), 2700(HY1), 2701(SA1), 2702(SU1), 2703(ST1), 2704(ST1), 2705(SU1), 2706(HY1), 2707(HY1), 2708(HY1), 2709(PO1), 2710(HY1), 2711(HY1), 2712(ST1), 2713(HY1), 2714(PO1), 2715(ST1), 2716(HY1), 2717(SA1), 2718(SA1), 2719(SU1)[c], 2720(SA1)[c], 2721(ST1), 2722(PP1)[c], 2733(PO1)[c], 2724(HY1)[c], 2725(ST1)[c].

FEBRUARY: 2726(WW1), 2727(EM1), 2728(EM1), 2729(WW1), 2730(WW1), 2731(EM1), 2732(SM1), 2733(WW1), 2734(SM1), 2735(EM1), 2736(EM1), 2737(PL1), 2738(PL1), 2739(PL1), 2740(PL1), 2741(EM1), 2742(ST2), 2743(EM1), 2744(WW1), 2745(WW1), 2746(SM1), 2747(WW1), 2748(EM1), 2749(WW1), 2750(WW1)[c], 2751(EM1)[c], 2752(SM1)[c], 2753(PL1)[c], 2754(ST2)[c], 2755(PL1).

MARCH: 2756(HY2), 2757(IR1), 2758(AT1), 2759(CO1), 2760(HY2), 2761(IR1), 2762(IR1), 2763(HY2), 2764(ST3), 2765(HY2), 2766(HW1), 2767(SA2), 2768(CO1), 2769(IR1), 2770(HY2), 2771(SA2), 2772(HY2), 2773(CO1), 2774(AT1), 2775(IR1), 2776(HY2), 2777(HY2), 2778(SA2), 2779(ST3), 2780(HY2), 2781(HY2)[c], 2782(HW1)[c], 2783(SA2)[c], 2784(CO1), 2785(CO1)[c], 2786(IR1)[c], 2787(ST3)[c], 2788(AT1)[c], 2789(HW1).

APRIL: 2790(EM2), 2791(SM2), 2792(SM2), 2793(SM2), 2794(SM2), 2795(SM2), 2796(SM2), 2797(SM2), 2798(EM2), 2799(EM2), 2800(EM2), 2801(EM2), 2802(ST4), 2803(EM2)[c], 2804(SM2)[c], 2805(ST4)[c].

MAY: 2806(SA3), 2807(WW2), 2808(HY3), 2809(WW2), 2810(HY3), 2811(WW2), 2812(HY3), 2813(WW2), 2814(HY3), 2815(WW2), 2816(HY3), 2817(HY3), 2818(SA3), 2819(WW2), 2820(SA3), 2821(WW2), 2822(WW2)[c], 2823(HY3), 2824(SA3), 2825(HY3), 2826(SA3)[c], 2827(HY3)[c].

JUNE: 2828(SM3), 2829(EM3), 2830(EM3), 2831(IR2), 2832(SM3), 2833(HW2), 2834(IR2), 2835(EM3), 2836(IR2), 2837(IR2), 2838(SM3), 2839(SM3)[c], 2840(IR2)[c], 2841(HW2)[c], 2842(EM3)[c], 2843(ST5), 2844(ST5), 2845(ST5), 2846(ST5)[c].

c. Discussion of several papers, grouped by divisions.

# AMERICAN SOCIETY OF CIVIL ENGINEERS

OFFICERS FOR 1961

## PRESIDENT

GLENN W. HOLCOMB

## VICE-PRESIDENTS

*Term expires October 1961:*
CHARLES B. MOLINEAUX
LAWRENCE A. ELSENER

*Term expires October 1962:*
DONALD H. MATTERN
WILLIAM J. HEDLEY

## DIRECTORS

*Term expires October 1961:*
THOMAS J. FRATAR
EARL F. O'BRIEN
DANIEL B. VENTRES
CHARLES W. BRITZIUS
WAYNE G. O'HARRA
FRED H. RHODES, JR.
N. T. VEATCH

*Term expires October 1962:*
ELMER K. TIMBY
SAMUEL S. BAXTER
THOMAS M. NILES
TRENT R. DAMES
WOODROW W. BAKER
BERNHARD DORNBLATT

*Term expires October 1963:*
ROGER H. GILMAN
HENRY W. BUCK
EARLE T. ANDREWS
C. MERRILL BARBER
JOHN D. WATSON
HARMER E. DAVIS

## PAST PRESIDENTS
*Members of the Board*

FRANCIS S. FRIEL

FRANK A. MARSTON

---

## EXECUTIVE SECRETARY
WILLIAM H. WISELY

## TREASURER
E. LAWRENCE CHANDLER

## ASSISTANT SECRETARY
DON P. REYNOLDS

## ASSISTANT TREASURER
LOUIS R. HOWSON

---

# PROCEEDINGS OF THE SOCIETY

HAROLD T. LARSEN
*Manager of Technical Publications*

PAUL A. PARISI
*Editor of Technical Publications*

MARVIN L. SCHECHTER
*Associate Editor of Technical Publications*

IRVIN J. SCHWARTZ
*Assistant Editor of Technical Publications*

---

## COMMITTEE ON PUBLICATIONS

THOMAS M. NILES, *Chairman*

WAYNE G. O'HARRA, *Vice-Chairman*

BERNHARD DORNBLATT

JOHN D. WATSON

HENRY W. BUCK

HARMER E. DAVIS